On The Optimality of Restricting Credit: Inflation-Avoidance and Productivity

ABSTRACT

To analyze a conflict in the use of credit, the paper presents a model in which the consumer uses up resources in order to avoid the inflation tax through the use of exchange credit. It determines optimal credit tax policy with a given inflation rate and then also allows the credit to also have productivity benefits. In an example economy without capital, the credit tax is optimal when the resource loss from credit use, dominates the productivity effect and the inefficiency of substitution towards leisure as a result of the credit tax. The paper also examines second- best inflation policy in this context, given a credit tax. Including physical capital, the paper extends the economy to an endogenous growth setting and shows how restricting inflation avoidance can have the interpretation of increasing the efficiency of human capital, while decreasing productivity. The effect on the growth rate and the role of leisure is discussed.

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Max Gillman*, Economics Institute, Academy of the Sciences of the Czech Republic, P.O.Box 882, Politickych veznu 7, 111 21 Prague 1, Czech Republic, Europe; email: max.gillman@cerge.cuni.cz; Tel: 420-2-2400-5194; Fax: 420-2-2422-7143;

and Department of Economics, Central European University; Nador u.9; 1051 Budapest, Hungary; Tel: 36-1-327-3225; Fax: 36-1-327-3232; email: econdept@ceu.hu.

1. Introduction

Roubini and Sala-i-Martin (1995) suggest that credit restrictions can be good for decreasing inflation-tax avoidance but bad for accumulating capital. In their model, the government controls the given level of financial intermediation, which negatively affects the consumer's utility value of money and positively affects the accumulation rate of the capital stock. An increase in intermediation decreases the demand for money and the inflation-tax proceeds. It can be optimal to repress intermediation despite a decrease in the growth rate.

Alternative approaches focus on inflation-tax avoidance by modeling intermediation endogenously through an exchange technology. In Schreft (1992) and Gillman (1993), the consumer chooses how many resources to devote to producing exchange (trade) credit, and so avoid the inflation tax. Serving only for tax avoidance, Schreft and Gillman (1987) find it socially optimal to eliminate the waste of resources in tax avoidance by prohibiting such avoidance activity. A restriction on avoidance may work at first but over time taxpayers tend to avoid such restrictions, as the existences of "non-bank" banks may illustrate. Using an explicit exchange technology and a particular example of a credit restriction, Lucas (1993) lets the consumer avoid reserve requirements in a way that uses up even more resources in total inflation-tax avoidance. The government restricts the first exchange alternative and the agent produces a second unrestricted exchange alternative, using a costlier technology than the first. The restriction thereby induces less efficient inflation-tax avoidance activity that can cause an even larger waste of resources in the total avoidance activity. Such literature further develops the issue of inflation avoidance, but leave unanswered the broader Roubini and Sali-i-Martin (1995) question about the tradeoff between the costs and benefits of credit use in a second-best world with the inflation rate given.[1] In particular, what if endogenous financial intermedation also furthers intertemporal accumulation?

This paper addresses the issue from a base model of intermediation that is endogenously produced in order to avoid the inflation tax. This further develops inflation analysis as part of the standard tax analysis of first and second-best optima (Lipsey and Lancaster, 1959). First, in exploring inflation-tax avoidance in the base model, the paper separates the bank and consumer problems[2] to show that the equilibrium relative price of the credit services is equal to the nominal interest rate net of the credit tax. In the context of the margins of substitution, the elasticities, and the inflation cost calibration, this explains why a second-best optimum can be a credit tax that is only as big as the nominal interest rate. Intuitively, the consumer as banker sets the marginal cost of credit service production (the marginal avoidance cost) equal to the credit service price. Making a tax on credit use as high as the marginal cost of production pushes the net supply price of credit services to zero, induces only money use, and deters all inflation-tax avoidance except goods-to-leisure substitution. If the latter effect is of small importance relative to the gain in resources from no credit use, then this is second-best optimal.

To include real benefits of the credit use, the paper initially introduces an economy-wide increase in the marginal product of labor from the average level of exchange credit use. With a trade-off comparable to Roubini and Sali-i-Martin (1995), the paper finds a range where restricting the credit is optimal only when the inflation rate becomes moderately high, and inflation-avoidance activity begins using up more resources than are generated by the higher marginal product of labor. This extends application of the theory of second best to money and credit markets, including a first-best optimum that

1. See Hartley (1998) for related work in which the credit good is specified through preferences.
2. See Aiyagari et al. (1995).

extends Friedman (1969). The example also shows how second-best credit-tax and inflation-tax policies depend on the nature of the economy, which is interpreted in terms of developed or developing.

The paper also discusses related results from the base model as extended to a Lucas (1988) endogenous growth model (see also Ireland, 1994, and Marquis and Reffett, 1995). A credit tax has only a level effect rather than a growth effect when there is no leisure use, and this effect is interpreted as increasing the efficiency of human capital. Positive leisure use implies a lower return on human capital and a lower growth rate; when inflation increases leisure, credit restrictions can increase the growth rate only by decreasing the leisure use. In Roubini and Sali-i-Martin (1995), in which there is no leisure, the growth rate changes because financial intermediation is assumed to increase the rate of change of the capital stock. With a different approach, Hartley (1998) shows how financial intermediation can affect the stock of capital rather than its growth rate, using information asymmetry and a credit constraint. Kiyotaki (1998) also uses a credit constraint, whereby the physical capital stock equals the collateral value of the intertemporal credit that is made available. Given a Kiyotaki- type effect, if exchange credit use leads to a higher collateral value being put on the intertemporal credit, then the capital stock ends up being higher. With this literature as motivation for making a somewhat weaker assumption than Roubini and Sali-i-Martin, the paper lastly considers restricting financial intermediation when it is assumed to affect the average level of the capital stock in the growth model.

2. The Economy with an Explicit Financial Intermediation Sector 2. The economy

Consider an economy with an explicit bank sector that produces credit services only for exchange transactions. And let there be a tax on exchanges made with credit. Exchange for goods occurs across a (0,1] continuum of stores as indexed by s.[3] The stores each sell a consumption basket

3. Excluding s=0 precludes a credit-only economy in which the price level is not well-defined.

or good that differs by color (as in Lucas, 1980). Let the good at store s and at time t be denoted by $c(s,t)$. This continuum can be thought of as existing across a geographical region where the average consumer values the different cultural dimensions of each region. Utility depends equally on each color of good, and induces "rainbow" consumption. The only other difference on the continuum is that the cost of exchange through private banking services gradually increases as the distance increases from the store indexed by $s=1$. With the lowest cost at $s=1$, this can be thought of as an international financial center; an index near zero can be thought of as isolated banks with a relatively high production cost for exchange services.

Let $P(t)$ denote the price of any good at any store at time t. In equilibrium, the good's price must be the same across all stores because it reflects only the production cost of the good, and each good is produced by the same linear, labor-only, technology, with w the given, constant marginal product of labor. The shadow price in contrast will differ across stores as this includes both the goods price of $P(t)$ and a shadow cost of exchange. Exchange induces a shadow cost because it is "self-produced" by the average agent who acts as either a consumer carrying around money or as a bank that produces the credit services. The shadow exchange cost is the same for all goods purchased by money and is just the foregone interest from carrying around money, $P(t)i(t)$, where $i(t)$ is the nominal interest rate. Since the production of credit services varies across the continuum, the shadow cost of credit varies with the store index. In particular, let the time per good that is used to produce credit services at store s be denoted by $t(s)$, and let this production time be given as a linear, decreasing, function of the store index s: $t(s)=A(1-s)$, with $A \in R_{++}$. From the equilibrium conditions, the shadow cost of exchange by credit at store s equals the value of the production time, or $wP(t)A(1\text{-}s)$.

If the financial intermediation activity is restricted in any way, then this can be thought of as raising the implicit price of purchasing a good with credit. Let this restriction be represented by an explicit tax that adds $T_e(t) \geq 0$ to the explicit goods price *P(t)*, making the price of credit goods $P(t) + T_e(t)$, where the tax proceeds are returned lump sum to the consumer through a transfer, denoted *G(t)*. Defining $t_e \equiv T_e(t) / P(t)$, so that t_e is constant for all time periods, the price of a good bought with credit at store *s* becomes $P(t)(1+t_e)$, while the total shadow price *s* is $P(t)[1+t_e+wA(1-s)]$, in the equilibrium marginal rates of substitution.

2.1 The Agent as a Money User

On the basis of the shadow costs of exchange of money versus credit, which are affected by the credit tax, the representative agent chooses the point in the continuum at which to switch between credit and money. Call this marginal store $\bar{s}(t)$, now another choice variable along with goods, leisure (also in the utility function below), and the money stock. The consumer uses credit where its shadow cost is below the nominal interest rate and this occurs for the relatively less expensive stores indexed from $\bar{s}(t)$ to *1*. More formally, the money goods are $c(s,t),\ s \in (0,\bar{s}(t)]$, and the credit goods are $c(s,t),\ s \in (\bar{s}(t),1]$.[4] The aggregate, or composite, money and credit goods are $\int_0^{\bar{s}(t)} c(s,t)ds$ and $\int_{\bar{s}(t)}^1 c(s,t)ds$. Letting *M(t)* denote the money stock at time t, the agent's demand for credit through the choice of $\bar{s}(t)$ gives rise to the model's particular form of the "Clower" or "cash" constraint:

$$M(t) \geq P(t)\int_0^{\bar{s}(t)} c(s,t)ds \qquad \textbf{(1)}$$

And the supply of money enters the economy through lump sum transfers of *V(t)*:

$$M(t+1) = M(t) + V(t) \qquad \textbf{(2)}$$

4. For comparison to the standard cash-in-advance economies with credit (as in Lucas and Stokey 1983, and Englund and Svennson 1988), each money good can be thought of as $c_1(s,t)$ on the lower part of the (0,1] continuum, and each credit

Assuming a constant inflation rate in order to derive a closed-form solution, define $V(t) \equiv pM(t)$ with $p \in R$, so that $M(t+1) = M(t)(1+p)$.

2.2 The Agent as Banker

The agent as banker in effect has "plants", "branches", or just say locations across the continuum, each with a differentiated cost and profit. Let the profit per unit of consumption at each location be denoted as $P(s,t)$, with total profit at location *s* equal to $P(s,t)c(s,t)$. Let the bank sell the credit service at any store for $P_F(s,t)$. In the competitive equilibrium below (equation (5)), $P_F(s,t) = P_F(t)$, for all *s*; the fee is the same across all stores. Costs at each location *s* are the marginal product *w* as factored by the labor hours used in the credit production, specified linearly as *A(1-s)*. Per unit profit at each location is given by

$$P(s,t) = P_F(t) - wP(t)A(1-s). \qquad \textbf{(3)}$$

The total profit across the continuum at locations at which the consumer uses credit is given by $\int_{\bar{s}(t)}^{1} \Pi(s,t)c(s,t)ds$; total revenues of the bank from all of its locations are $P_F(t)\int_{\bar{s}(t)}^{1} c(s,t)ds$; and total costs are $wP(t)\int_{\bar{s}(t)}^{1} A(1-s)c(s,t)ds$. With technology linear for each store, there is a greater time required per good at the locations that are added on the margin when the range of the use of credit spreads on the continuum. This gives the bank's aggregate operations an increasing marginal cost of expanding the range of credit use through a lower $\bar{s}(t)$. Given $P_F(t)$ and $c(s,t)$, the bank's profit problem is to find the marginal location for production of credit services:

$$\underset{\bar{s}}{Max} \int_{\bar{s}}^{1} \Pi(s,t)c(s,t)ds = P_F(t)\int_{\bar{s}}^{1} c(s,t)ds - wP(t)\int_{\bar{s}}^{1} A(1-s)c(s,t)ds. \qquad \textbf{(4)}$$

The first-order condition gives that the relative price of finance equals the value of the time required at

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good as $c_2(s,t)$, on the upper part of the store continuum.

the marginal store $\bar{s}(t)$ at which the bank offers credit; this can be thought of as the marginal (time) cost of the credit service:

$$P_F(t) / P(t) = Aw(1 - \bar{s}(t)). \qquad \textbf{(5)}$$

This implies that bank profits are zero at the marginal location $\bar{s}(t)$, which is used to substitute in for $P(\bar{s},t)$ in the consumer's first-order condition with respect to $\bar{s}(t)$. At every other store *s*, bank profits in equilibrium are $P(s,t)= P_F(t) - wP(t)A(1-s)$, which is used to substitute in for $P(s,t)$-$P_F(t)$ in the consumer's first-order condition with respect to the credit good, $c(s,t)$, $\bar{s}(t) < s \leq 1$, in order to determine that the shadow cost of a credit good is $P(t)[1 + t_e + wA(1 - s)]$.

2.3 The Agent as Consumer

The consumer receives the profit from the banking activity as part of income. The other earned income is from working for wages in supplying labor for goods production and for banking. This equals the marginal product of labor *w* factored by the total time endowment of *1* less leisure, denoted *x(t)*, or $w[1-x(t)]$. The consumer also receives the lump sum money supply transfer *V(t)*, and the lump sum transfer of the credit tax revenues *G(t)*, where in equilibrium,

$$G(t) = t_e P(t) \int_{\bar{s}(t)}^{1} c(s,t)ds \qquad \textbf{(6)}$$

Also assume there is an exogenous real asset endowment of $a \in R_+$. The expenditures on goods using credit equal $P(t)(1+t_e)\int_{\bar{s}(t)}^{1} c(s,t)ds$, and the purchases with money equal $P(t)\int_0^{\bar{s}(t)} c(s,t)ds$. The consumer also pays $P_F(t)\int_{\bar{s}(t)}^{1} c(s,t)ds$ for the credit services. The net income equals the change in money holdings between periods, $M(t+1)-M(t)$. With a market discount rate of $q \equiv 1/(1+i)$, the discounted stream of nominal asset value is

$$\sum_0^t q^t \Big[M(t+1) - M(t) - \int_{\bar{s}(t)}^1 \Pi(s,t)c(s,t)ds - wP(t)[1-x(t)] - V(t) - P(t)a$$
$$- G(t) + P(t)\int_0^1 c(s,t)ds + [P_F(t) + t_e P(t)]\int_{\bar{s}(t)}^1 c(s,t)ds \Big] = 0 \qquad \textbf{(7)}$$

The utility function, with $a \geq 0$, is the log example: $U(t) \equiv \int_0^1 (\ln[c(s,t)] + a \ln[x(t)])ds.$ Given the preference discount factor b, the consumer maximizes the discounted utility with respect to $c(s,t), 0 < s \leq \bar{s}; c(s,t), \bar{s} < s \leq 1; x(t); M(t+1); \bar{s}(t)$, subject to equations (1), and (7), with $l(t)$ and m the multipliers. The full solution, given in Appendix A, requires the consumer first-order conditions, the market clearing conditions (2), and (3), the bank's first-order condition (5), and the government constraint (6), with the restriction that $t_e \leq i < Aw$ so that $\bar{s} \in (0,1]$. Also, in lieu of an explicit bond market, to save on notation, assume the Fisher equation of $q = b/(1+p)$.

Given that the bank profit is zero at the marginal store $\bar{s}(t)$, the first-order condition with respect to $\bar{s}(t)$ plus other first-order conditions implies that the relative price of the credit service in equilibrium equals the nominal interest rate net of the credit tax:

$$P_F(t)/P(t) = i - t_e. \qquad \textbf{(8)}$$

3. Leveling the inflation distortion through credit taxes

Along the "external" $\bar{s}(t)$ margin, the bank and consumer problems imply that $Aw(1-\bar{s}(t)) = P_F(t)/P(t) = i - t_e$. The solution is $\bar{s}(t) = \bar{s} = 1 - (P_F(t)/P(t))/Aw$. The credit tax pushes down the net supply price of credit so that the credit supply and its use falls to zero while $\bar{s}$ goes to one. This does not indicate the second-best optimum, but this and other parts of the equilibrium indicate the effect of the credit tax in a way suggestive of the optimum.

3.1 The Margins of substitution

The credit tax also raises the consumer's shadow price of using credit. From equations (A.1-A.5) of Appendix A, the marginal rate of substitution between any money good and a credit good at store s, equals the ratio of the relative shadow prices, or $(1+i)/(1+t_e+Aw(1-s))$. Raising t_e to i makes the cost of credit greater than the interest rate at all stores in the (0,1] s-index continuum, and so fully offsets the inflation distortion towards using credit. With $i = t_e$ and only money usage, the shadow price of goods consumption equals $(1+i)$ for all goods, and the shadow cost of goods relative to leisure equals $(1+i)/w$. This ratio is higher than when some credit is used at a lower exchange cost than i; the credit tax induces substitution towards leisure. Because the first-best $(i=0)$ optimum has a comparable ratio of $1/w$, this substitution goes in the wrong direction in terms of efficiency, and leaves unclear what is the second-best optimum.

3.2 The Elasticities

The relative impact of the different effects of the credit tax, on money versus credit use and on goods versus leisure use, can be seen to some extent within the interest elasticity of money demand, denoted $h_i^{M/P}$:

$$h_i^{M/P} = -\left(\frac{i}{Aw}\Big/\left(1-\frac{i-t_e}{Aw}\right)\right) - \frac{i}{1+i} - \frac{i[(2+i)(i-t_e)-Aw]/[Aw(1+i)^2]}{\left[1+a-\frac{i}{1+i}\left(1-\frac{(i-t_e)}{Aw}\right)-\frac{t_e}{Aw}\ln\left(\frac{1+i}{1+t_e}\right)\right]} \quad \textbf{(9)}$$

The credit tax primarily affects the first term which equals $-(1-\bar{s})/\bar{s}$ and shows the extra-marginal effect of substitution between money and credit for the purchase of any good. This tends to dominate the second term, $i/(1+i)$, which is the effect of the money-good to leisure substitution (see Gillman, 1993) and is unaffected by the credit tax. The third term is the interest elasticity of the marginal utility of income. It includes the effects of the change in tax revenues from inflation and from the credit tax, and from the change in income as a result of a change in leisure, and it has a negligible magnitude for moderate inflation rates.

This suggests that the main effect of the credit tax, rather than increased substitution from goods to leisure is decreased substitution from money to credit for the purchase of any good. Through this substitution in the first term, the credit tax decreases the magnitude of the interest elasticity. A corollary view comes through computation of the elasticity of substitution between the money and credit inputs (to exchange). Defining money as $\int_0^{\bar{s}(t)} c(s,t)ds$, credit as $\int_{\bar{s}(t)}^{1} c(s,t)ds$, and the relative price of money to credit as i/Aw, the elasticity of substitution between money and credit, denoted s, is

$$s \equiv \left[\partial \left(\frac{\int_{\bar{s}(t)}^{1} c(s)ds}{\int_0^1 c(s,t)ds} \right) \Big/ \partial \left(\frac{i}{Aw} \right) \right] \left[\frac{i}{Aw} \Big/ \frac{\int_{\bar{s}(t)}^{1} c(s)ds}{\int_0^1 c(s,t)ds} \right] = \left[\frac{i}{Aw} \Big/ \left(1 - \frac{i - t_e}{Aw} \right) \right] + \frac{i}{1+i} \qquad \textbf{(10)}$$

The first two terms of the interest elasticity, denoted $\hat{h}$, equal the negative of the elasticity of substitution between money and credit: $-\hat{h} = s$. And $\partial s / \partial t_e < 0$ (for $t_e \leq i < Aw$, so that $\bar{s} \in (0,1]$). The credit tax unambiguously decreases the elasticity of substitution between money and credit, and the interest elasticity of money demand with marginal utility held constant. By Bailey (1956)-type logic that positively links the magnitude of the money demand elasticity to the magnitude of welfare cost estimate, this increased inelasticity from the credit tax lowers the welfare cost of a given inflation.[5] And equation (9) suggests that the Bailey result, in which he ignores changes in the marginal utility of income, can be more precisely stated as a positive link between the magnitude of the elasticity of substitution between money and credit and the welfare cost inflation. These results show that the substitution effect of the credit tax towards more leisure and less goods ends up affecting only the marginal utility of income and is of less importance for cases of moderate sustained inflation. The dominant effect of the credit tax is left as the efficient gain back to society of the resources spent avoiding the inflation tax.

3.3 The Second-Best Credit Tax Given the Inflation Rate

A way to determine the net welfare effect of a small increase in the credit tax and the second-best optimum is to compute a function giving the welfare cost of a given inflation rate, for variable credit tax rates. To derive such a welfare cost, substitute the equilibrium goods and leisure from Appendix A into the utility function and set the utility level in the equilibrium (with i given and $a=0$) equal to the utility level at the first-best optimum (with i=0). Then solve for a from this equation. With $v(\bullet)$ denoting the indirect utility function, this equation is $v(i,a,t_e,\bullet) = v(0,0,t_e,\bullet)$. Normalizing by Beckerian (1965) "full" income of $1 \cdot w$ (the time endowment equals 1), and given $\bar{s} = 1-(i-t_e)/(Aw)$, the cost function is

$$\frac{a}{w} = \left\{ \left[\left(\frac{1+i}{1+t_e} \right)^{\left(\frac{1+t_e}{Aw}\right)} \Big/ \frac{e^{(1-\bar{s})}}{(1+t_e)} \right]^{\frac{1}{1+a}} \left[1 - \frac{1}{1+a} \left(\frac{i\bar{s}}{1+i} + t_e \ln\left(\frac{1+i}{1+t_e} \right) \Big/ Aw \right) \right] \right\} - 1 \qquad \textbf{(10)}$$

The first bracketed term contains the effect of substitution among good and leisure, and the second bracketed term contains the effect of the marginal utility of income, including the tax transfers. Specifying the parameters as $Aw=0.54$, $a = 2.27$, and $b = 1/1.03$ (Gillman, 1993), the graph of equation (10) in Figure 1 illustrates how the welfare cost locus monotonically shifts down towards its minimum as t_e rises and approaches i. The ridge of optimality in Figure 1 at $t_e = i$ shows the second-best optimal locus. To indicate in Figure 1 the infeasibility of $t_e > i$ where $\bar{s}$ >1, welfare costs are set equal to zero for $t_e > i$.

3.4 Sensitivity and Robustness of the Results

The second-best result of $t_e = i$ is robust to any specification of $a < \infty$ and Aw<0, given the

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5. The Bailey link is shown to be valid in Gillman (1995) for general equilibrium as well as for partial equilibrium.

restriction of $Aw > i - t_e$ that keeps the $\bar{s}$ within range. As a goes to zero, the credit tax has induces a proportionately larger gain in welfare. For large values of a, the gain from a credit tax is proportionately less. At a =0, the first-best and the second-best optimum are the same, since in this case the credit tax induces no inefficient goods-to-leisure substitution. As *Aw* goes to its minimum allowable value, the benefit of the credit tax is proportionately greater; as *Aw* approaches infinity, the gain from the credit tax goes to zero. The credit tax is most important when leisure preference is small and the cost of credit low. The actual value of the leisure preference tends to be calibrated at low levels, near 2, and the technological advance in the banking sector suggests that the cost of credit appears to be falling. If the productivity in credit production outpaces the economy-wide productivity increase (*A* falls faster than *w* rises), then the cost of credit will trend downwards and the gain from the credit tax will rise.

With this log-utility example, leisure demand depends on its own shadow price *w*, a, and the marginal utility of income (see Appendix A). More generally, leisure can also depend positively on the shadow price of the credit good, including the credit tax. Then the credit tax induces more leisure and it is possible that $i = t_e$ may not be second-best. Further, when the credit tax causes additional negative income results, it can be sub-optimal. Destruction of the credit tax revenues, as occurs typically with a regulation rather than an explicit tax, can make the credit tax decrease welfare, as can a cost of tax collection (Gillman, 1987). These results compare to the sub-optimality of the reserve requirements in Lucas (1993) where there is even less efficient tax avoidance activity. With log-utility, and aside from such negative income effects, the optimality of an effectively prohibitive credit restriction is robust to alternative forms of the restriction. It is second-best optimal to set to infinity a tax on time in credit activity (like an income tax on the bank sector profits), and to set to zero a ceiling on the interest rate that credit funds can earn (see Gillman, 1987). However introducing a benefit to credit use changes the optimum from restricting all inflation-tax avoidance to trading off such restrictions against fewer benefits.

4. Restricting Credit When it Has Other Benefits4. Restricting the Benefits of Credit-Induced Productivity Enhancement

Many attribute an increase in the capital stock to the development of the financial intermediation sector. This generally would increase the economy's marginal product of labor. In the labor-only economy of section 2, this can be captured by assuming that the marginal product of labor depends linearly on the degree to which credit is used on average in the economy. In particular, let $w(t) = w(\bar{s}_a, t)$, where $w_1(\bar{s}_a, t) < 0$. The example used is $w(s_a) = \hat{w} + n(1 - \bar{s}_a)$, where $\hat{w}, n \in R_+$. In equilibrium, $\bar{s} = \bar{s}_a$; the agent recognizes the external effect of average credit use in solving the equilibrium.[6] The marginal rate of substitution between any money good and a credit good at store *s* now equals $(1 + i)/[1 + t_e + A(\hat{w} + n[1 - \bar{s}])(1 - s)]$. The credit tax still directly raises the shadow price of credit goods but also now indirectly lowers the shadow price through the term that contains the fraction of stores using credit $(1 - \bar{s})$.

4.1 The theory of second best and the optimum quantity of money

The construction of a welfare cost function again allows the net effect of the tax to be determined, although now this is less standard because of the externality. The planner solves for the real goods endowment a from the utility equation $v(i,a,t_e,\bullet) = v(0,0,t_e,\bullet)$ and the normalized welfare cost of inflation $a/w(\bar{s}_a)$ is given in Appendix A. The Friedman optimum of $i=0$ no longer by itself gives the minimum of this function because of the marginal external benefit of credit use. It is still part of the first-best optimum, as the marginal social cost of money is still equal to zero. Now, in addition, the marginal

6. See Lucas (1988) for a similar treatment of the effect of external human capital.

social cost and marginal social benefit of credit must be equalized. This marginal social cost equals the marginal private cost, which equals the supply price of credit, or the nominal interest rate net of the credit tax. The marginal social benefit is the rate of gain that comes from the economy-wide productivity increase. Call this rate *b*, and then the first-best optimum occurs when $i=0$ and $i-t_e=b$, or when $t_e=-b$. This means that the first-best optimum includes credit subsidization through a negative credit tax rate. The rate *b* varies with the economies parameters, especially n. In the second-best optimum *b* also varies with t_e and *i*. When the nominal interest rate is given at a rate above the Friedman optimum, say at $\bar{i}$, then the second-best optimum is achieved with a credit tax rate of $t_e=\bar{i}-b(\bar{i})$. When the credit tax rate is given at a rate above its first-best subsidy rate, say at $\bar{t}_e$, then the second-best optimum is achieved with a nominal interest rate of $i=\bar{t}_e+b(\bar{t}_e)$.

4.2 The Results

The results are shown both in Figure 2 and Table 1. In Figure 2, $n=0.09$, $a=2.27$, $A=1$, $\hat{w}=1$, and $b=1/1.03$. The first-best is the minimum of the cost function in Figure 2 at $i=0$, and $t_e=-0.107$ ($\bar{s}=0.895$). The external benefit causes the cost function to lose its monotonicity with respect to the interest rate and the credit tax rate. When $\bar{i}>b(\bar{i})$, credit taxes are beneficial up to the second-best rate of $t_e=\bar{i}-b(\bar{i})$. For $\bar{i}<b(\bar{i})$, credit subsidies are beneficial up to a rate of $-t_e=-(\bar{i}-b(\bar{i}))$. Figure 2 shows the trough along which lies the minimum welfare cost locus that corresponds to the optimal credit tax for each given level of the interest rate, and the optimal interest rate for each given level of the credit tax. Table 1 shows how the results vary with parameter values. Changes in the external factor n cause a smaller effect as the leisure preference parameter a rises. Marginal changes is A and $\hat{w}$ have little effect.

Consider a case that might apply to industrial nations. Let the inflation rate be set by policy at zero. With a rate of time preference of $(1/b)-1 \equiv r = 0.03$, the nominal interest rate is i=0.03. Because of its developed state, let the external factor n be relatively low at n=0.03, and let a =2.27 as calibrated for the U.S. (Gillman, 1993). Then the optimal credit tax is zero (not reported in Table 1). But if there is a greater external effect of credit use, then the implied policy is a subsidization of credit use, perhaps through government supply of bank and capital market oversight, business practice codification, and even insurance coverage. Or consider being give a moderately high inflation rate in a developed economy, say with i=0.10, n=0.03, and a =2.27. A credit tax 6.7 percent would be optimal because it would deter excessive avoidance of the inflation tax (see Table 1). Lastly, consider a case that might apply to developing or transition countries. Let the implicit tax rate on credit use be given at 0.05 because of inefficient government control of intermediation. With emerging credit markets, the external effect might be relatively high, say with n=0.06. With a =2.27, and $r = 0.03$, the optimal interest rate is about 0.07, implying an inflation rate of about four percent (see Table 1). With other examples of the specification of $w(\bar{s}_a)$, it is conceivable that there are multiple ranges of the inflation tax in which credit taxes or subsidies are efficient.

5. Endogenous Growth and the Credit Tax

An endogenous growth setting distinguishes between the growth and level effects of the credit taxation. Assume the same production of credit services using only labor and let credit use initially allow avoidance of the inflation tax without other benefits. Assume a Cobb-Douglas production of goods with physical and human capital, denoted k and h, as in Lucas 1988 but without an external effect from average human capital: $y = Bk^{g}(lh)^{1-g}$, with $B \in R_{++}$, $g \in (0,1)$. Time spent in human capital accumulation is the endowment of one minus labor time, denoted l, leisure, and time spent in providing credit services: $1 - l - x - \int_{\bar{s}}^{1} A(1-s)c(s)ds$. With c denoting the sum of all consumption goods ($c \equiv \int_{0}^{1} c(s,t)ds$), along the balanced growth path, the growth rates of y, c, M/P, k, and h are the same and the levels of c/k, c/h, h/k, x, l, and $\bar{s}$ are stationary. With f the efficacy factor of human capital investment, and r the rate of time preference, the growth rate of consumption g, is $g=f-r$. See Appendix B for these and the following results.

5.1 Without Leisure

When the preference parameter for leisure is zero, the growth rate g is unaffected by the inflation rate. And it is unaffected by the credit tax. Time is used up in avoiding the inflation tax by increasing the production of credit services. But rather than this credit time being taken away from human capital production, it comes one-for-one from goods production because the return on human capital and time invested in human capital remain unchanged. Increases in the money supply growth rate cause a rise in the inflation rate and a decrease in the amount of goods produced for consumption and investment: labor hours l falls, the ratio c/h falls, and the ratio h/k rises. In a sense, inflation induces a less efficient use of human capital. The imposition of a credit tax t_e offsets this distortion. The credit tax leaves unchanged the growth rate of consumption, causes l and c/h to rise, and h/k to fall, and so induces a lessor waste of tax-avoidance time and a greater production of goods. The maximum c/h occurs at $t_e = i$ similar to the optimality results in the section 2 model.

5.2 With Leisure

With a positive preference for leisure (x), the return on human capital investment becomes $f(1-x)$. The more time that the agent puts into leisure, the less is the return on the human capital investment. The growth rate of consumption becomes $f(1-x)-r$ and correspondingly falls when leisure increases.

The inflation rate equals s-[f(1-x)-r] and rises as leisure rises. An increase in t_e that decreases leisure will raise the growth rate of consumption, lower the inflation rate, and increases c/k. However since the credit tax increases substitution from goods to leisure in the base model of sections 2-4, a decrease in leisure may be unlikely; a topic for further research.

5.3 With an effect of intermediation on capital

Now consider letting the capital stock k rise with the average degree of financial intermediation, 1-$\bar{s}_a$, so that $k=k(\bar{s}_a)$ and $k'(\bar{s}_a)<0$. For example with $k(\bar{s}_a) = k(2-\bar{s}_a)$, k is unchanged when (1-$\bar{s}_a$)=0 (only money use), and k is doubled when (1-$\bar{s}_a$)=1 (only credit use). An increase in the average extra-marginal credit use raises the capital stock in equilibrium, where $\bar{s}_a = \bar{s}$ and the other equilibrium conditions are unchanged. In the economy without leisure, the balanced path growth rate is unchanged, as are the ratios c/k and hl/k. Other variables and ratios are effected and the full balanced-path solution requires numerical techniques. The effect of an external capital stock increase because of greater credit use also depends upon the transitional dynamics. However it can be shown that a small increase in credit use decreases the amount of labor hours when the economy's credit market is "emerging" (a nominal interest rate near to zero, or a credit tax rate near to the nominal interest rate.) In this case, the level of human capital rises with the external increases in the capital stock. And the level of productivity, in terms of the ratio of goods to labor hours c/l, rises as a result of the expansion of credit use. This gives a sense in which the external increase in labor productivity in the labor-only model of section 4 can be thought of as an abstraction of this more involved economy. Both models involve a tradeoff between productivity benefits of credit use and lost resources in inflation-tax avoidance, which a credit restriction must balance in the optimum. With the insight of the endogenous balanced growth path, the tradeoff now can be characterized as weighing the human capital inefficiency that comes from inflation tax avoidance against the productivity benefits of capital enhancement. With positive levels of leisure, the tradeoff affects the growth rate.

6. Conclusion

Focusing initially on inflation-tax avoidance through the use of exchange credit, the paper finds a robustness of the second-best credit restrictions in the face of a given non-optimal inflation. In a cash-in-advance model with an explicit bank sector, the paper presents excise-type taxes on credit goods that need to be of a level proportional to the interest rate in order to achieve a second-best prohibitive optimum that is characterized by a uniform "rainbow" consumption across the continuum. It derives the market price of the credit service and uses this to build intuition for this result on the margins, through elasticities, and in a welfare cost function. It then includes a positive external effect of credit use on the marginal product of labor and shows a locus of second-best optimal money and credit taxes that balance inflation-tax avoidance and productivity effects. The intuition of the optima is extended to an endogenous growth model. Given that inflation effects the growth rate through its effect on leisure, the paper suggests determining in further research whether restricting credit use eliminates wasteful credit use but causes an even greater use of leisure and a lower growth rate. Related research might incorporate an intertemporal function for exchange credit through a Kiyotaki (1998)- type credit constraint.

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APPENDIX A: BASE MODEL: First-order conditions and equilibrium solution

First-order conditions:

$$[b^t / c(s,t)] - b^t\, l(t)P(t) = 0, \text{ for } 0 < s \le \bar{s}(t); \qquad \textbf{(A.1)}$$

$$[b^t / c(s,t)] - m\,q^t\, P(t)[wA(1-s) + 1 + t_e\,] = 0, \text{ for } \bar{s} < s \le 1; \qquad \textbf{(A.2)}$$

$$b^t \int_0^1 [a\ / x(t)]ds - m\,q^t\, P(t)w = 0; \qquad \textbf{(A.3)}$$

$$-b^t\, l(t)P(t)c(\bar{s},t) + m\,q^t\, P(t)c(\bar{s},t)[wA(1-\bar{s}) + 1 + t_e] = 0\,; \qquad \textbf{(A.4)}$$

$$b^t\, l(t) - m\,q^{t-1} = 0 \qquad \textbf{(A.5)}$$

Solution:

$c^*(s) = (w + a) / \left[(1+i)\left(1 + a - [1/(1+i)][1 - (i - t_e)/(Aw)] - t_e[\ln(1+i) - t_e \ln(1+t_e)]/(Aw)\right)\right]$

for $0 < s \le \bar{s}(t)$; $\bar{s} = 1 - [(i - t_e)/Aw]$; $l(t)P(t) = 1/c^*$;

for $\bar{s}(t) < s \le 1$, $c(s,t) = (1+i) / \{[1 + t_e + Aw(1-s)]\, l(t)P(t)\}$;

$x = a\ / [w\, l(t)P(t)]$; $M(t)/P(t) = \bar{s} / [l(t)P(t)]$, or

$$\frac{M(t)}{P(t)} = \frac{(1 - [(i - t_e)/(Aw)])(w + a)}{\left[(1+i)\left(1 + a\ - [1/(1+i)][1 - (i - t_e)/(Aw)] - t_e[\ln(1+i) - t_e \ln(1+t_e)]/(Aw)\right)\right]} \qquad \textbf{(A.6)}$$

Productivity effect of credit use:

With $w(\bar{s}) = \hat{w} + n(1-\bar{s})$

$c^* = [a + \hat{w} + n(1-\bar{s})] / \left[(1+i)\left(1 + a - (i\bar{s}/[1+i]) - t_e \ln[(1+i)/(1+t_e)] / (A[\hat{w} + n(1-\bar{s})])\right)\right]$;

$\bar{s} = 1 - (\hat{w}/2n) + [(\hat{w}^2/4n^2) + (1 - t_e)/(An)]^{0.5}$; the rest of the solution is then as above.

Welfare cost function:

$$\frac{a}{w(\bar{s})} = \left\{\left[\frac{\left([1+i]/[1+t_e]\right)^{\left(1+\frac{1+t_e}{A(\hat{w}+n[1-\bar{s}])}\right)}}{e^{(1-\bar{s})}[1 + n(1-\bar{s})/\hat{w}]/(1+t_e)}\right]^{\frac{1}{1+a}}\left[1 - \frac{1}{1+a}\left(\frac{i\bar{s}}{1+i} + \frac{t_e \ln[(1+i)/(1+t_e)]}{A(\hat{w} + n[1-\bar{s}])}\right)\right]\right\} - 1\,.$$

APPENDIX B: ENDOGENOUS GROWTH

The bank problem here is made implicit as in Gillman (1993) (rather than explicit as in section

2), with no effect on the first-order conditions, and continuous time is used, both for ease of presentation. Let d denote the nominal sum of the money and capital assests, let r and w denote the marginal products of capital and effective labor, let the utility function be modified for balanced growth, and let $1/(1+r) \equiv b$. Human capital investment function depends linearly on time spent in human capital accumulation: $\dot{h} = mf\left[1 - l - x - \int_{\bar{s}}^{1} A(1-s)c(s)ds\right]h$, where $f \in R_+$ is the efficacy parameter.

The following Hamiltonian extends the base model:

$$\underset{c_1(0<s\leq\bar{s}),c_2(\bar{s}<s\leq1),\bar{s},M,k,d,h,l}{Max} \; \boldsymbol{H} = \int_0^{\infty} e^{-rt}\left[\ln\int_0^1 c(s)ds + b(x^{1-q}-1)/(1-q)\right]dt$$

$$+l\left[Pkr + Plhw - P\int_0^{\bar{s}} c(s)ds - P(1+t_e)\int_{\bar{s}}^1 c(s)ds + V + G + \dot{P}k\right]$$

$$+mfh\left[1 - l - x - \int_{\bar{s}}^1 A(1-s)c(s)ds\right] + g\left[d - M - Pk\right] + d\left[M - P\int_0^{\bar{s}} c(s)ds\right].$$

The solution:

$\dot{c}/c = g = f - r$; $r = f$; $w = (1-g)(k/lh)^g$; $lh/k = (f/g)^{1/(1-g)}$; $c/k = (f/g) - g$;
$\bar{s} = 1 - [(i - t_e)/Aw]$;
$l = [1 - g/f]\Big/\left[1 + (c_1/k)(k/lh)(1+i)[(1+t_e)\ln([1+t_e]/[1+i]) + i - t_e]/Aww\right]$;
`;
$c/h = (c/k)(k/hl)l$.

The credit tax t_e affects only l and c/h. Given the restrictions in section 2 ($t_e \leq i < Aw; s \in (0,1]$), the following comparative statics can be shown:
$\partial l/\partial i < 0; \partial l/\partial t_e > 0; \partial(c/h)/\partial i < 0; \partial(c/h)/\partial t_e > 0; \partial(k/h)/\partial i < 0; \partial(k/h)/\partial t_e > 0$.
With leisure (x), $\dot{c}/c = g = f(1-x) - r$; $r = f(1-x)$; $lh/k = (f[1-x]/g)^{1/(1-g)}$; and $c/k = (f[1-x]/g) - g$.

TABLE 1: THE OPTIMAL POLICIES UNDER DIFFERENT PARAMETERS

Optimal Credit Tax t_e					***Optimal Nominal Interest Rate i***				
a =0					=0				
	A=1	A=1	A=1	A=.10		A=1	A=1	A=1	A=2
i=	0	0.5	0.10	0.10	t_e =	-0.05	0	0.05	0.05
n=					n=				
0.00	0.00	0.05	0.10	0.10	0.00	0.0*	0.0	0.0	0.0
0.03	-.030	0.021	0.071	0.071	0.03	0.0*	0.029	0.079	0.079
0.06	-.057	-.007	0.043	0.042	0.06	0.007	0.057	0.107	0.107
0.09	-.083	-.033	0.016	0.015	0.09	0.033	0.084	0.134	0.134
a =2.27					=2.27				
	A=1	A=1	A=1	A=.10		A=1	A=1	A=1	A=2
i=	0	0.5	0.10	0.10	t_e	-0.05	0	0.05	0.05
n=					n=				
0.00	0.00	0.05	0.10	0.10	0.00	0.0*	0.0	0.0	0.0
0.03	-.029	0.019	0.067	0.067	0.03	0*	0.018	0.05*	0.05*
0.06	-.056	-.010	0.036	0.037	0.06	0.005	0.037	0.069	0.05*
0.09	-.082	-.038	0.007	0.011	0.09	0.023	0.057	0.090	0.063

* The optimum is a boundary point constrained either by $i \geq 0$ or $\bar{s} \in (0,1]$.