Applied and Computational Mathematics Seminars 2017-2018

All seminars are held at 15:10 in Room M/2.06, Senghennydd Road, Cardiff unless stated otherwise.

Programme organiser and contact: Dr Usama Kadri

Date	Speaker	Seminar
3 October 2017	Dr. Davide Crivelli (School of Engineering, Cardiff University)	**Detecting damage with waves: an overview of structural health monitoring research in Engineering** Acoustic emission is a promising technique for monitoring damage on structures. It provides a non-invasive, real-time and passive system for detecting cracks and defects in a range of structures as they happen. Applications range from civil structures, such as bridges and railways; rotating machinery; aerospace, such as impact and damage detection in advanced composite structures. We face multiple challenges however when the amount of data from an Acoustic Emission system has to be interpreted in a reliable way. Wave propagation, source characterization, interaction with damage and sensor characteristics make traditional signal interpretation and source localization a challenging task. This talk will give a general overview of acoustic emission for structural health monitoring. It will present a series of case studies, describing how Cardiff research has tackled the main challenges of signal localization and interpretation, and how the research has helped bringing the technology closer to commercial applications. It will also present a collaboration with Maths stemmed from acoustic emission research and applied to low frequency acoustic gravity waves in the search of MH370. Bio Sketch: Davide Crivelli is a Lecturer at Cardiff School of Engineering. He obtained his PhD in Mechanical Engineering at Politecnico di Milano, with a dissertation on structural health monitoring. He then continued his research on aerospace damage detection in the Cardiff University School of Engineering, where he is now a lecturer. He teaches in a range of mechanical design modules, and was part of the 2017 Formula Student UK winner Cardiff Racing team. His research deals mainly with processing and interpretation of high frequency time series data, to characterise and interpret source severity and location in aerospace applications. His research interests span from signal fingerprinting techniques to monitor the health status of bearings and rotating machinery, real time analysis of composite curing status during resin infusion, monitoring of drilling tool status during repeated operations, and anything

		involving data reduction and information extraction from long time series signals. http://www.cardiff.ac.uk/people/view/364357-crivelli-davide
10 October 2017	Dr. Thomas Woolley (School of Mathematics, Cardiff University)	**Patterns, cellular movement and brain tumors** I present three pieces of work that illustrate the power of mathematics as a tool for understanding biology. Although the applications appear to be disparate the underlying mathematics is very similar. I begin by looking at theoretical and experimental pattern formation, with emphasis on whisker formation in mice. Here, reaction-diffusion equations are used to provide insights into how the wavelength of the whiskers are controlled. Next, I consider the phenomena of blebbing cells. Initially, I use a diffusion equation to understand the motion of muscle stem cells and illustrate how old cells fundamentally move differently to old cells. This is then extended to include solid mechanics, which allows us to link the structural properties of the cell to their motion. Finally, reaction-diffusion equations are used to understand the formation of brain tumours. Critically, the cells move at different speeds in white and grey matter, including this information can lead to very different migration patterns of the tumours. Bio Sketch: Dr Thomas Woolley studied mathematics at University of Oxford between 2004-2017. Through his education he ended up specializing in mathematical biology, where his doctorate focused on understanding the pattern formation behind fish spots and zebra stripes. Alongside this research he now investigates mathematical models of stem cell movement. The hope is that by understanding how stem cells move we can influence them and, thus, speed up the healing process. When not doing mathematics he is a keen participant in mathematical outreach workshops and has given a variety of popular maths lectures nationally and internationally. He has previously worked for the BBC, illustrated Marcus du Sautoy's book and worked on the popular maths show "Dara O'Briains school of hard sums". Most recently he was the Fellow of Modern Mathematics at the London Science Museum and is helped redesign their mathematics gallery.
18 October 2017	Scott Morgan (School of Mathematics, Cardiff University)	**Stability of Oscillatory Rotating Disk Boundary Layers** The rotating disk boundary layer has long been considered as providing an archetypal model for studying the stability of three-dimensional boundary-layer flows, and the crossflow inflexion point instability mechanism is common to both the rotating disk boundary layer and the flow over a swept wing. Thus the investigation of strategies for

controlling the behaviour of disturbances that develop in the rotating disk flow may prove to be helpful for the identification and assessment of aerodynamical technologies that have the potential to maintain laminar flow over swept wings.

We will consider the changes in the stability behaviour that arise when the rotating disk base-flow configuration is altered by imposing a periodic modulation in the rotation rate of the disk surface. Thomas et. al. [Proc. R. Soc. A (2011) 467:2643-2662] have previously demonstrated that Tollmien-Schlichting waves can be stabilised when a similarly induced Stokes layer is conjoined to a plane channel flow.

Current work encompasses three distinct investigatory approaches. Linearised direct numerical simulations have been conducted, using the vorticity-based methods that were first adopted by Davies & Carpenter [J. Comput. Phys (2001) 172:119-165]. These simulations are complemented by a local in time linear stability analysis, that is made possible by imposing an artificial frozen base-flow approximation. This localised analysis is deployed together with a more exact global treatment based upon Floquet theory, which avoids the need for any simplification of the temporal dependency of the base-flow.

24 October 2017	Václav Klika, PhD (Czech Technical University)	**On modelling self-organisation in real systems**

Nowadays models for self-organisation are being used in systems with a great degree of complexity and across disciplines. We show that the widely used Turing model is sensitive to inputs, type of domain growth, but also to the precision of model formulation itself. Hence a great care is needed when applying Turing's model for self-organisation to real problems. For this purpose we consider derivation of evolution equations within non-equilibrium thermodynamic to identify physically relevant formulations. Only then we subject these models to a detailed mathematical analysis. We offer possible extensions of the concept of self-organisation to more general situations and discuss its physical interpretation.
The essence and importance of these ideas is illustrated on the reaction-diffusion-advection system, where we indicate that such a system should be preferred from both physical and mathematical viewpoint. Further we point to the indispensable role of physical viewpoint during relevant model formulations. Using the non-equilibrium thermodynamic framework physically consistent extensions of Turing model are revealed as well as functional constraints for present parameters.

Bio Sketch:

		Václav Klika finished his PhD in 2009 at the Czech Technical University in Prague where in 2016 he received a permanent position (Associate prof; formerly was appointed as an assistant prof. since 2010). Since his PhD he spent several short termed visiting postdocs abroad (OCCAM, ETH, Zaragoza). His research interests are mainly in applied mathematics (mathematical biology) and non-equilibrium thermodynamics.
7 November 2017	Richard Hewitt, PhD (University of Manchester)	**Localised streaks in a Blasius boundary layer** Streaks are common feature of perturbed boundary-layer flows. They play a central role in transient growth mechanisms and are a building block of exact coherent structures. Most theoretical work has focused on streaks that are periodic in the spanwise direction, but in this work we consider a single spatially localised streak embedded into a Blasius boundary layer. For small streak amplitudes, we show the perturbation can be described in terms of a set of eigenmodes that correspond to an isolated streak/roll structure. These modes are new, and arise from a bi-global eigenvalue calculation; they decay algebraically downstream and may be viewed as the natural three-dimensional extension of the two-dimensional Libby \& Fox (1963, JFM vol. 17) solutions. Despite their bi-global nature, we show that a subset of these eigenmodes is fundamentally related to both the Libby & Fox solutions, and those presented by Luchini (1996, JFM vol. 327), as derived for (spanwise) periodic disturbances at small spanwise wavenumber. This surprising connection is made by an analysis of the far-field decay of the bi-global state. We also address the downstream development of nonlinear streaks, confirming that the aforementioned eigenmodes are recovered as the streak/roll decays downstream. Some comparisons are made with available experimental data.
14 November 2017	John A. Mackenzie, PhD	**An Adaptive Moving Mesh Method for Geometric Evolutions Laws and Bulk-Surface PDEs: Application to a Model of Cell Migration and Chemotaxis** In this talk I will consider the adaptive numerical solution of curve-shortening flow with a driving force. An adaptive moving mesh approach is used to distribute the mesh points in the tangential direction. This ensures that the resulting meshes evolve smoothly in time and are well adjusted to resolve areas of high curvature. Experiments will be presented to highlight the improvement in accuracy obtained using the new method in comparison with uniform arc-length mesh distributions. We will also discuss the use of the evolving adaptive curve mesh in the adaptive generation of bulk meshes for the solution of bulk-surface PDEs in time dependent domains. The main motivation for developing these computational tools is the modelling of single cell migration and

		chemotaxis. Chemoattractant gradients are usually considered in terms of sources and sinks that are independent of the chemotactic cell. However, recent interest has focused on “self-generated” gradients, in which cell populations create their own local gradients as they move. Here we consider the interplay between chemoattractants and single cells. To achieve this we model the breakdown of extracellular attractants by membrane-bound enzymes. Model equations are parameterised using published estimates from Dictyostelium cells chemotaxing towards cyclic AMP. We find that individual cells can substantially modulate their local attractant field under physiologically appropriate conditions of attractant and enzymes. This means the attractant concentration perceived by receptors can be a small fraction of the ambient concentration. This allows efficient chemotaxis in chemoattractant concentrations that would be saturating without local breakdown. Bio Sketch: Having graduated from the University of Strathclyde, I completed my DPhil at the University of Oxford (under the supervision of Professor K.W. Morton) on “Cell Vertex Finite Volume Methods for the Compressible Navier-Stokes Equations”. I continued at Oxford on a postdoctoral research funded by the EU working on a posteriori error estimation for finite volume approximations of hyperbolic PDEs with Professor E. Suli. I was appointed to a lecturing position at Strathclyde in 1993 and I’m currently a Reader. Since 2008 I have worked with Professor R. Insall at the CRUK-funded Beatson Institute for Cancer Research. Our interest is in the use of mathematical and computational techniques to understand the mechanisms of cell migration and chemotaxis. My main contribution in this area is in the development of adaptive moving mesh techniques to solve partial differential equations on evolving domains.
21 November 2017	Professor Alexander Korobkin (University of East Anglia)	**Diffraction of hydroelastic waves by a circular cylinder** Linear problem of wave diffraction is studied for a circular cylinder mounted at the sea bed and piercing the fluid surface which is covered by ice plate of infinite extent. The water depth is constant. The ice plate is modeled by a thin elastic plate of constant thickness clamped to the surface of the cylinder. One-dimensional incident hydroelastic wave of small amplitude propagates towards the cylinder. and is diffracted on the cylinder. Deflection of the ice plate and the bending stresses in it are determined by two methods: (a) using the integral Weber transform in radial direction, (b) using the vertical modes for the fluid of constant depth with the rigid bottom and elastic upper boundary. The solution by the second method is straightforward but we cannot prove that the solution is complete because the properties of the vertical modes are not known yet. The solution by the Weber transform is more complicated but

		this solution is unique. In this talk we will show that these two solutions are identical. This result justifies the method of the vertical modes in the hydroelastic wave diffraction problems.
27 November 2017	Dr Eldad Avital (Reader School of Engineering and Materials Science Queen Mary University of London)	**Challenges in Fluid-Structure Interaction from Renewable Energy Application to Bio-Fluids** In the first part of the talk we will briefly describe a recently developed computational methodology coupling fluid-flow simulation using finite volume with structural dynamics simulation using a combined finite-discrete element method. Applications in water flow as of sediments and hydro-kinetic turbines will be discussed, followed by looking at recent designs for hydrogen fusion power and the challenges. Bio-fluid applications of red blood cell flow and upper renal system will also be illustrated. In the second part of the talk we will discuss our aerodynamic method for blade design calling for continuous surface curvature. It will be computationally and experimentally shown to increase aerodynamic efficiency, particularly at high incidence where stall delay can be achieved. Reduction of tonal noise for low Reynolds number blade sections will also be addressed. Biosketch: Dr Avital is a Reader in Aero-Mechanical Engineering at Queen Mary University of London. He received his PhD in Aerospace Engineering from University of London (1998), MSc in Mechanical Engineering (Tel Aviv 1993), BSc Aerospace Engineering (Technion 1988). His research focuses on fluid dynamics and acoustics, and interaction with structural dynamics; ranging from the fundamentals to applications in water flow engineering, renewable energy, power production and bio-fluid devices as blood pumps. His research has been supported by research councils, EU, dstl and SMEs. He largely collaborates with colleagues in Europe, USA, Canada, Brazil, Israel, India, China and South Korea
12 December 2017	Andrey Melnik, PhD (University of Glasgow)	**Constitutive modelling of myocardium and other fibre-reinforced soft tissues** Nonlinear solid mechanics is used to model normal function and pathological conditions in soft tissues. In many cases, the mechanical behaviour of tissues can be adequately represented using an idealised elastic material. For instance, myocardium can be regarded as a passive (non-linear) hyperelastic solid with pronounced anisotropic properties due to its complex microstructure. In the first part of the talk we examine basic ideas utilised in structural and semi-structural constitutive models, discuss the multiplicative decomposition framework for growth and remodeling (G&R), and highlight some challenges in using these elements.

The second part of the talk is dedicated to the Generalised Structure Tensor (GST) approach, which is used to formulate constitutive models for anisotropic fibre-reinforced composited with fibre distribution or dispersion [Gasser et al. JRS'06]. The GST approach has been so far successfully applied to models based on invariants I4 and I5, which capture the effect of deformation on each fibre family in isolation. We extend the GST approach to models based on the invariant I8, which couples two fibre families. Using the Holzapfel-Ogden model for myocardium, we demonstrate that accounting for fibre dispersion in the I8 term can have a significant effect on the predicted material response and may also reduce material symmetry.

19 December 2017

Prof. Till Bretschneider (University of Warwick)

Image-based modelling of cell dynamics

Modern live-cell fluorescence microscopy enables us to visualise dynamic cellular processes in unprecedented detail. I will present ongoing research projects which are concerned with bringing together i) image analysis methods for tracking cells and their movements as well as quantifying spatio-temporal patterns of fluorescently labelled cellular constituents, and ii) mathematical models to investigate regulatory mechanisms of cellular biochemistry and mechanics.

16 January 2018

Professor Frederic Dias (University College Dublin)

What makes ocean waves go rogue in the real world?

The study of extreme ocean waves is a rapidly expanding area of research worldwide. Although much work in this area is based on modeling and experiments in controlled wave tanks, the starting point of all studies is wave observation in the natural world. During this talk, we will provide some evidence of extreme wave events, describe the main mechanisms for their generation and conclude with what we believe makes ocean waves go rogue in the real world.

Frederic Dias received a PhD in Civil and Environmental Engineering from the University of Wisconsin, Madison, USA, in 1986. He started his career in the US before coming back to France to join CNRS in 1990. In 2000, he moved to Ecole Normale Superieure de Cachan and has been a Professor of Applied Mathematics since. In 2009, he went to University College Dublin (UCD) on leave to work on wave energy converters. He is now leading the wave group at UCD. In 2012, he received an advanced grant from the European Research Council (ERC) to work on extreme wave events. In 2014, he received a proof of concept grant from the ERC to work on wave measurement. His stay at UCD has been extended until 2019.

23 January 2018	Georges Limbert (University of Southampton (UK) and University of Cape Town (South Africa))	**A mechanistic approach to skin biophysicsLearning from mathematical and computational models** Besides the brain, no other organ of the human body plays such a central role in our everyday biological and social life than the skin. After all, this interface is the first line of defence of our body against the external environment and acts as a physical interface. It controls many types of exchanges between our inner and outside worlds which take the form of mechanical, thermal, biological, chemical and electromagnetic processes. Moreover, the skin tells a story about our health, age, past traumas, emotions, ethnicity, and our social and physical environments. Considering the place of the skin in our life and its multiple physiological functions, understanding its complex physiology and biophysics in health, disease and trauma has become, particularly in the last two decades, a broad multidisciplinary research arena. To unravel some of the secrets of such a complex organ new experimental, imaging and computational techniques are needed and novel mechanistic theories explaining particular mechanobiological processes need to be formulated and put to the test. Developing and exploiting such an integrated framework underpin many aspects of our research which aims to understand the interplay between the microstructural and material properties of the skin, particularly as they evolve over the life course. As mounting evidence suggests, the skin microstructure can play a critical role in how macroscopic deformations are modulated at the microscopic level. These structural mechanisms are also at the heart of skin tribology by being part of, and conditioning mechanical load transmission and the nature of surface physics interactions. Skin biophysics is therefore fundamental to many industrial sectors from biomedical devices, personal care and cosmetic products to vehicle safety, textile, sport equipment, wearable electronics and tactile surfaces. In this talk, I will present some of the latest modelling approaches we develop to gain a mechanistic understanding of the interplay between the material and structural properties of the skin, and ultimately, to exploit this knowledge for a variety of clinical and industrial applications. Examples will include computational contact homogenisation procedures to study skin friction, constitutive modelling of skin ageing and analysis of skin surface instabilities to understand mechanisms of wrinkle formation (Figure 1).
30 January 2018	Raphael Stuhlmeier (University of Plymouth)	**Evolution of statistically inhomogeneous degenerate water wave quartets** Nonlinear interaction, along with wind input and dissipation, is one of the three mechanisms which drive wave evolution, and is included in every modern wave–forecast model. The mechanism behind the nonlinear interaction terms in such models is based on the kinetic equation for wave spectra

derived by Hasselmann. This does not allow, for example, for statistically inhomogeneous wave fields, nor for the modulational instability which depends on such inhomogeneity, and which has been implicated in the appearance of exceptionally high rogue waves.

Beginning with the basics of third-order wave theory, we sketch the derivation of a discretized equation for the evolution of random, inhomogeneous surface wave fields on deep water from Zakharov's equation, along lines first laid out by Crawford, Saffman, and Yuen. This allows for a general treatment of the stability and long-time behaviour of broad-banded sea states. It is investigated for the simple case of degenerate four-wave interaction, and the instability of statistically homogeneous states to small inhomogeneous disturbances is demonstrated. Furthermore, the long-time evolution is studied for several cases and shown to lead to a complex spatio-temporal energy distribution. The possible impact of this evolution on the statistics of rogue wave occurrence is explored within the framework of this simplified example.

Biosketch: Raphael joined the Centre for Mathematical Sciences at Plymouth University in 2017 after several years at the Faculty of Civil Engineering at Technion - Israel Institute of Technology. He received his PhD in mathematics from the University of Vienna, Austria. His research focuses on various aspects of the mathematics of water waves.

6 February 2018

Rosemary Dyson (University of Birmingham)

Fibre-reinfoced fluids: from plants to extracellular matrix and beyond

Many biological systems depend on an underlying mechanical anisotropy to give the system required functional properties. This anisotropy is often created via fibres embedded within a ground matrix. For example cellulose microfibres within plant cell walls which enable directional pressure driven expansion and collagen fibres within extracellular matrix which guide cell behaviour. Similar ideas can be exploited within a synthetic biology context to investigate the properties of biological molecules via spectroscopy. We employ a common mathematical framework to study these diverse problems, which we discuss here.

13 February 2018

Kostas P. Soldatos (University of Nottingham)

On the theory of fibre-reinforced materials: past, present and future

The nonlinear theory of fibre-reinforced materials originated at the middle of the 20th century through pioneering attempts of Rivlin and Adkins to model mathematically large elastic deformation of rubber-like articles used commonly in industry, such as pneumatic tyres and fire hose. Today the use of the theory is spread worldwide and, apart from its usefulness in common industrial applications, it also enables modelling and understanding of complicated biological processes, including the behavior and growth of soft and hard biological tissue. This talk is naturally biased by the fact that the theory was essentially nurtured and, for long after the aforementioned pioneering attempts, developed through the research activity and effort of a group of Nottingham-based mathematicians. It aims to outline, briefly and lightly, theoretical features and principal relevant concepts that became known gradually over the years, assisted, and continue to assist the development of the theory. The foundation of the talk is accordingly based on the initial non-polar, and the more recent polar hyperelastic versions of the theory. However, attention is also paid to the the fact that, in the small deformation regime, the non-polar hyperelastic version of the theory reduces naturally to its older linear anisotropic elasticity counterpart, which has been, and is still used extensively in the static and dynamic analysis of advanced fibre-reinforced structural composites. If time allows, relevant modelling developments that concern plastic behaviour of fibre-reinforced solids and fluid-like behavior of fibre-reinforced resins will also be referred to.

Biosketch: Dr Kostas (Konstantinos) P. Soldatos joined the permanent academic staff of the University of Nottingham in 1990. He had previously held visiting/postdoc positions in the Department of Theoretical Mechanics, University of Nottingham, U.K. and the Department of Civil Engineering, Carleton University, Ottawa, Canada, as well as a permanent academic position in the University of Ioannina, Greece, where he also received his first degree (Mathematics) and his PhD (Mechanics). He has a long track of highly cited research publications and has given invited talks and seminars in eleven different countries. His research interests and activity span a number of interconnected subjects and areas of continuum Solid Mechanics and its applications, including lineal and non-linear theory of isotropic and anisotropic elasticity, theory of plasticity, theory of mass-growth, theoretical modelling and characterization of composite materials, as well as modelling and analysis of thin-walled structures and structural components (beams, plates and shells).

6 March 2018

Prof. Alain Goriely (Mathematical Institute, University of Oxford)

The mathematics and mechanics of brain morphogenesis

The human brain is an organ of extreme complexity. Its intricate folded shape has fascinated generations of scientists and has, so far, defied a complete description. How does it emerge? How is its shape related to its function? In this talk, I will review our current understanding of brain morphogenesis and its unique place within a general mathematical theory of biological growth. In particular, I will present simple models for basic pattern

		formation and show how they help us understand brain folding and skull formation.
13 March 2018	Prof. Marco Marletta (School of Mathematics, Cardiff University)	**An inverse problem in electromagnetism with partial data** Magnetic induction tomography involves determining the coefficients in the time-harmonic Maxwell equations (the permeability and permittivity) from surface measurements of current and voltage on a body. For practical reasons these measurements are made only on a part of the boundary. In this talk we show that partial boundary measurements uniquely determine the Maxwell system. This work is part of a joint project with Malcolm Brown (COMSC) and Paul Ledger (Swansea School of Engineering).
1 May 2018	Dr. Katerina Kaouri (School of Mathematics, Cardiff University)	**Mathematical modelling for calcium signalling and for water resources management** Calcium signalling is one of the most important mechanisms of information propagation in the body. In embryogenesis the interplay between calcium signalling and mechanical forces is critical to normal embryonic development, but poorly understood. Several types of embryonic cells exhibit calcium-induced contractions and experiments indicate that calcium oscillations and contractions are linked via a two-way feedback mechanism. I will discuss some experiments and appropriate deterministic mathematical models that I have developed with Philip Maini and Jon Chapman (Oxford) and (the experimentalists) Paris Skourides (Univ. of Cyprus) and Neophytos Christodoulou (Cambridge). I will finish the math-bio part of my talk with stochastic models for calcium signalling that I have developed with Ruediger Thul (Nottingham) and Ioannis Lestas (Cambridge). Furthermore, I will briefly present a water management project I have been working on with collaborators in Cyprus, UK and India. In collaboration with the Cyprus Water Development Department, we have been working on improved recharge protocols for the Germasogeia aquifer supplying water to Limassol, Cyprus. Furthermore, for the case of accidental contamination, we developed a model with which we can predict the extent and speed of contamination in the aquifer for various scenarios of practical interest. This project emerged from the 1st Study Group with Industry in Cyprus.
15 May 2018	Scott Morgan (School of Mathematics, Cardiff University)	**Stability of Oscillatory Rotating Disk Boundary Layers** The rotating disk boundary layer has long been considered as providing an archetypal model for studying the stability

of three-dimensional boundary-layer flows, and the crossflow inflexion point instability mechanism is common to both the rotating disk boundary layer and the flow over a swept wing. Thus the investigation of strategies for controlling the behaviour of disturbances that develop in the rotating disk flow may prove to be helpful for the identification and assessment of aerodynamical technologies that have the potential to maintain laminar flow over swept wings.

We will consider the changes in the stability behaviour that arise when the rotating disk base-flow configuration is altered by imposing a periodic modulation in the rotation rate of the disk surface. Thomas et. al. [Proc. R. Soc. A (2011) 467:2643-2662] have previously demonstrated that Tollmien-Schlichting waves can be stabilised when a similarly induced Stokes layer is conjoined to a plane channel flow.

Current work encompasses three distinct investigatory approaches. Linearised direct numerical simulations have been conducted, using the vorticity-based methods that were first adopted by Davies & Carpenter [J. Comput. Phys (2001) 172:119-165]. These simulations are complemented by a local in time linear stability analysis, that is made possible by imposing an artificial frozen base-flow approximation. This localised analysis is deployed together with a more exact global treatment based upon Floquet theory, which avoids the need for any simplification of the temporal dependency of the base-flow.