## **Seminars**

All seminars are held at 14:10 in Room M/2.06, Senghennydd Road, Cardiff unless stated otherwise.

The programme organiser and contact is Dr Suresh Eswarathasan.

	Date	Abstract
Sofia Lindqvist (Oxford)	23 April 2018	Counting zeros of quadratic forms in few variables
		Let Q(x_1,x_2,,x_n) be a quadratic form with integer coefficients. We are interested in counting the (asymptotic) number of zeros of Q. If the number of variables is at least 5 this can be done by using the HardyLittlewood circle method. In order to deal with n=4 one can use the Kloosterman method, and by using the so-called delta method one can go all the way down to n=3. I will give an introduction to the various methods used to answer this question, with a particular focus on the difficulties that arise when the number of variables is small.
Julio Andrade (Exeter)	23 April 2018	A Problem Involving Divisor Functions  In this talk, I will discuss and present some new results related to divisor functions. We will give a complete solution to a problem about the maxima of the pairwise divisor function, which goes back to Erdös. This is joint work with my PhD student Kevin Smith.

	Date	Abstract
Francesco Fanelli (Lyon)	16 April 2018	Asymptotic dynamics of non-homogenous fluids in fast rotation
		In this talk we consider a class of singular perturbation problems for systems of PDEs related to the dynamics of geophysical fluids. We are interested in effects due to both density variations and Earth rotation, and to their interplay. We specialize on the 2-D non-homogeneous incompressible Navier-Stokes equations with Coriolis force: our goal is to characterize the asymptotic dynamics of weak solutions to this model, in the limit when the rotation becomes faster and faster. If the initial density is a small perturbation of a constant state, we prove that the limit dynamics is essentially described by a homogeneous Navier-Stokes system with an additional forcing term, which can be seen as a remainder of density variations. If, instead, the initial density is a small perturbation of a truly variable reference state, we show that the final equations become linear, and moreover one can identify only a mean motion, described in terms of the limit vorticity and the limit density fluctuation function; this issue can be interpreted as a sort of turbulent behaviour of the limit flow. This talk is based on a joint work with Isabelle Gallagher.
Julien Barré (U. Nice)	9 April 2018	A very singular drift-diffusion equation and magneto-optical trap modelling

Joint work with Dan Crisan and Thierry Goudon (INRIA and University of Nice)

30 years ago, atomic physicist Jean Dalibard discovered the presence of a long range attractive force in laser-cooled atomic clouds, the so-called « shadow effect ». A simplified modelling of this effect yields a non linear drift-diffusion equation, which bears some similarities with Keller-Segel system, while being much more singular. I will show how precise estimates allow to overcome this strong singularity of the interaction, and to obtain global existence of a solution for large enough diffusion. However, the long time behavior of a solution for small diffusion is still an open question.

Radu Ignat (Toulouse)

9 April 2018

A DeGiorgi type conjecture for minimal solutions to a nonlinear Stokes equation

The aim is to study the symmetry of transition layers in Ginzburg-Landau type functionals for divergence-free maps in R^N. Namely, we determine a class of nonlinear potentials such that the minimal transition layers are one-dimensional. In particular, this class includes in dimension N=2 the nonlinearities w^2 with w being an harmonic function or a solution to the wave equation, while in dimension N>2, this class contains a perturbation of the standard Ginzburg-Landau potential as well as potentials having N+1 wells with

prescribed transition cost between the wells. For that, we develop a theory of calibrations for divergence-free maps in R^N (similar to the theory of entropies for the Aviles-Giga model when N=2). This is a joint work with Antonin Monteil (Louvain, Belgium).

Elena Issoglio (U. Leeds)

19 March 2018

SDEs, BSDEs and PDEs with distributional coefficients

In this talk I will present three families of differential equations (SDEs, BSDEs and PDEs) and their links to each other. The novel fact is that some of the coefficients are generalised functions living in a fractional Sobolev space of negative order. I will discuss the appropriate notion of solution for each type of equation and show existence and uniqueness results. To do so, I will use tools from analysis like semigroup theory, pointwise products, theory of function spaces, as well as classical tools from probability and stochastic analysis. The link between these equations will play a fundamental role, in particular the results on the PDE are used to give a meaning and solve both the forward and the backward stochastic differential equations. The main part of the talk will be about a semilinear parabolic PDE and related semilinear BSDEs and SDEs. If I have time, I will talk about some work in progress on a parabolic non-linear

	Date	Abstract
		PDE with a term which is quadratic in the gradient of the solution.
Ahmed Jama (Cardiff University)	12 March 2018	Generalised translations and periodic sets with applications to the Grushin plane  In this talk we introduce a new notion of
		translations, namely generalised translations along vector fields, and an associated notion of periodicity for sets, which apply to very general geometrical structures. We are in particular interested in applying these notions to the case of Grushin spaces. We also prove a Poincare inequality for an unbounded periodic set in this setting.
Michael Levitin	26 February 2018	Asymptotics of the sloshing and Steklov eigenvalues in planar domains with corners
(Reading)		I'll describe sharp eigenvalue asymptotics for the sloshing and Steklov problems in curvilinear polygons. The sloshing results prove a long standing conjecture of Fox and Kuttler. This is a joint work (partially in progress) with Leonid Parnovski, Iosif Polterovich and David Sher.
Ned Nedialkov (McMaster U.)	12 February 2018	Simulating Lagrangian Mechanics Directly
(iviciviastoi U.)	2010	We integrate numerically a system in a Lagrangian form directly, without deriving the underlying equations of motion explicitly. From a C++ specification of a

Lagrangian function and algebraic constraints, our "Lagrangian" facility applies automatic differentiation to prepare a differential-algebraic equation (DAE) system, which is then solved by our highindex differential-algebraic equation (DAE) solver DAETS. Lagrangian equations of the first kind contain algebraic constraints, resulting in an index-3 DAE; Lagrangian equations of the second kind are constraint-free, resulting in a system of ordinary differential equation (ODEs). The former are usually much simpler and easier to construct (in particular when using Cartesian coordinates) than the latter, which typically involve angle coordinates and non-trivial transformations to eliminate constraints. However, integrating an index-3 DAE is substantially more difficult than integrating an ODE. DAETS solves a highindex DAE as easily as an ODE. We model and simulate rigid-body mechanisms -mechanical systems with linked rigid parts and possible other parts such as springs -from a constrained Lagrangian formulation and using Cartesian coordinates. As a result, we have compact models and avoid lengthy symbolic transformations that are typically applied to derive a system of ODEs. We illustrate by examples in 2D (such as the Andrews Squeezer Mechanism, one of the MBS Benchmark problems) and 3D, and report results of

	Date	Abstract
		numerical solution by this method, with animations.
Lucia Scardia (U. Bath)	12 February 2018	The equilibrium measure for a nonlocal dislocation energy
		In this talk I will present a recent result on the characterisation of the equilibrium measure for a nonlocal and anisotropic energy arising as the Gamma-limit of discrete interacting dislocations, and an extension to more general anisotropies.
		This is joint work with J.A. Carrillo, J. Mateu, M.G. Mora, L. Rondi and J. Verdera.
Trevor Wooley (Bristol	5 February 2018	Efficient congruencing as p-adic decoupling
University)		We discuss the efficient congruencing approach to estimating mean values of exponential sums. This approach may be viewed as a p-adic analogue of the 1^2-decoupling approach of Bourgain, Demeter and Guth, and indeed the latter can be viewed as a real analogue of efficient congruencing. Both approaches prove the Main Conjecture in Vinogradov's Mean Value Theorem, and there are many generalisations and applications of these ideas. In this talk we will explain the background, give an idea of what underlies

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## **Abstract**

the method, and emphasise the viewpoint of efficient congruencing as a p-adic decoupling method. In this p-adic approach, one sees that such complications as the use of multi-linear Kakeya inequalities in the method of Bourgain et al. are no longer needed, and indeed the nature of the decoupling is particularly clean and intuitive. Despite the appearance of the word "p-adic", little by way of number theory is required, and we will aim to make the talk accessible to a fairly general audience.

Jon Bennett (U. 29 January Birmingham) 2018

The Brascamp—Lieb inequality in harmonic analysis

The Brascamp—Lieb inequality simultaneously generalises many important inequalities in analysis, such as the multilinear Holder, sharp Young convolution, and Loomis—Whitney inequalities. The purpose of this talk is to describe certain extensions of the Brascamp—Lieb inequality that have recently come to the fore in harmonic analysis and its applications.

Jiqiang Zheng 15 January (U. Nice) 2018

Dynamics of energy-supercritical nonlinear Schrodinger equation

In this lecture, I will talk about some basic mathematical problems in nonlinear PDEs, especially the dynamics of energy-supercritical nonlinear Schrodinger equation. First, we use compactness method to show the global existence of weak solutions.

Second, we utilize the concentration-compactness method to prove the "critical norm conjecture". Finally, we introduce two types of blowup solutions.

Peter Varju (University of Cambridge) 11 December2017

Recent progress on Bernoulli convolutions

The Bernoulli convolution with parameter lambda in (0, 1) is the measure nu\_lambda on the real line that is the distribution of the random power series sum of +/-\lambda^n, where +/- are independent fair coin tosses. These measures are natural objects from several points of view including fractal geometry, dynamics and number theory. The main question of interest is to determine the set of parameters for which the measure is absolutely continuous with respect to the Lebesgue measure, a problem that goes back to the 1930's.

If lambda < 1/2, then nu\_lambda is always singular being supported on a Cantor set. In the range lambda in [1/2, 1), there are examples for both type, nu\_lambda may be absolutely continuous or singular. Which parameters exhibit which behaviour is still not fully understood. In the last few years, our knowledge dramatically improved

	Date	Abstract
		thanks to the work of several authors and a new method based on the growth of entropy of measures under convolution. I will survey this recent progress.
Benjamin Texier	4 December 2017	Nonlinear stabilization
(Jussieu)	2017	For autonomous ordinary differential
		equations in finite dimensions, the
		Lyapunov stability theorem states that for sufficiently regular vector fields, linear stability implies nonlinear stability and linear instability implies nonlinear instability.
		The stability theorem generalizes to infinite dimensions. Generalizations of the
		instability theorem to infinite dimensions are known only under additional spectral or regularity assumptions.
		Which conditions are necessary for linear stability to imply nonlinear instability? In other words: can we find examples of
		flows which are linearly unstable but nonlinearly stable?
		This is joint work with Thierry Gallay (Grenoble) and Kevin Zumbrun (Indiana).
	4 December 2017	Spectral bounds for the torsion function and torsional rigidity.
		We discuss bounds for the torsion function and its L^1 norm (the torsional rigidity) for

	Date	Abstract
		an open set in Euclidean space with finite measure. Partly joint work with Vincenzo Ferone, Carlo Nitsch, Cristina Trombetti.
Yves Capdebosq	27 November 2017	Stability results for parabolic models in mathematical biology
University)	(Oxford University)	In this talk I will speak about stability results related to nonlinear parabolic equations (and, time permitting, systems of equations) that appear in models for cell growth. This is joint Work with Luca Alasio and Maria Bruna.
Shane Cooper	13 November	A general framework for the
(Durham University)	2017	homogenisation of high-contrast problems
Olliversity)	versity)	In homogenisation theory, the important questions of error estimates and (the very much related) spectral convergence are fraught with difficulties: one challenge is to establish the correct limit with respect to some topology that ensures spectral convergence, then one must handle the fact that the limits are in general defined in different function spaces to the prelimit.
		These issues appear because conventionally one first establishes a convergence result then asks questions of error estimates and/or spectral convergence. The problem is we first passed to the limit!

In this talk we present a novel framework to study the asymptotic behaviour of (a large class of) second-order linear elliptic PDE systems with periodic coefficients whose ellipticity constant degenerates in the limit of small period. We determine, under one-or-two readily verifiable assumptions, the leading-order behaviour of the (variational) solution with respect to the (small period) parameter. Error estimates, uniform in right-hand-side, are readily deduced in this process of determination.

Amongst other things, this work explains the differences between the convergence results/spectral asymptotics of classical and high-contrast homogenisation problems documented in literature. Additionally, we are compelled to revisit the central concept of averaging over the periodic reference cell in homogenisation theory. I shall present an example in the context of Magnetic-Schrodinger equations where this notion is not applicable.

This is joint work with Ilia Kamotski (UCL) and Valery Smyshlyaev (UCL).

Yulia Ershova (U. Bath)

6 November 2017

A unified operator-theoretical approach to high-contrast homogenisation in models of chain-type graphs

I am going to demonstrate how the new abstract unified approach to the problems

## **Abstract**

of high-contrast homogenisation developed in cooperation with Kiselev, Cherednichenko, and Naboko works in rather simple models of graphs periodic along one axis (chain-type graphs). I will consider three examples of such graphs which show that the above-mentioned new approach is indeed free from the crucial additional assumption that the stiff component of the media is to be connected. The result which our approach yields comprises not only the operator asymptotics in norm-resolvent topology but also sheds new light on the intricate relationship between the norm-resolvent limits of high-contrast media and the corresponding models of time-dispersive media.

Asma Hassannezad (University of Bristol) 30 October 2017 Higher order Cheeger type inequalities for the Steklov eigenvalues

In 1970 Cheeger obtained a beautiful geometric lower bound for the first nonzero eigenvalue of the Laplacian in term of an isoperimetric constant. Inspired by the Cheeger inequality, Cheeger type inequalities for the first nonzero Steklov eigenvalue have been studied by Escobar, and recently by Jammes. The generalization of the Cheeger inequality to higher order eigenvalues of the Laplacian in discrete and manifold settings has been studied in recent years. In this talk, we study the higher-order Cheeger type

inequalities for the Steklov eigenvalues. It gives an interesting geometric lower bound for the k-th Steklov eigenvalue. It can be viewed as a counterpart of the higher order Cheeger inequality for the Laplace eigenvalues, and also as an extension of Escobar's and Jammes' results to the higher order Steklov eigenvalues. This is joint work with Laurent Miclo.

Michael Magee (Durham University)

23 October 2017

Uniform spectral gap in number theory

I'll begin with Selberg's eigenvalue conjecture. This is an analog of the Riemann hypothesis for a special family of Riemann surfaces that feature heavily in number theory, for example in Wiles' proof of the Taniyama-Shimura conjecture. I'll explain how in the last 10-15 years, number theorists have had to turn to Anosov dynamics to obtain the approximations to Selberg's conjecture that became relevant to emerging 'thin groups' questions about Apollonian circle packings and continued fractions. I will explain what my contributions to this area were. Finally if I have time, I'll explain how I am pushing these techniques into the setting of Teichmuller dynamics in the pursuit of yet more interesting number theory questions.

	Date	Abstract
Erez Nesharim	9 October 2017	The t-adic Littlewood conjecture is false
(University of		

Littlewood conjecture and the p-adic Littlewood conjecture are famous open problems in number theoy. Their positive characteristic analogues are well known. In a joint work with Faustin Adiceam and Fred Lunnon we show that the so called "t-adic" Littlewood is false over \$F\_3((1/t))\$. Our counter example is concrete, but the proof uses computers. It is based on a generalisation of Dodson's condensation algorithm for computing the determinant of a matrix.

Baptiste 2 Octobe Morisse (Cardiff University)

York)

2 October 2017 Well-posedness for systems of PDEs in Gevrey regularity Abstract

Systems of first-order, quasilinear PDEs form a wide and interesting topic. Many physical equations which arise in hydrodynamic are of this form (Euler equation, KdV, Burgers...). I will first introduce the basic notions for the study of such systems, give examples and then point out what are the borderline cases - which we call weakly hyperbolic or weakly elliptic systems. In order to deal with such cases, I will explain the importance of working in spaces of highly regular functions: the Gevrey spaces.