Deformations of Quantum Field Theories on Spacetimes with Killing Vector Fields

Eric Morfa-Morales
ESI, Vienna

based on joint work with Claudio Dappiaggi and Gandalf Lechner

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Outline

Warped convolution on Minkowski spacetime

Algebraic deformation method for quantum field theories:
[Grosse,Lechner:07,08] [Buchholz,Summers:08] [Buchholz,Lechner,Summers:10]

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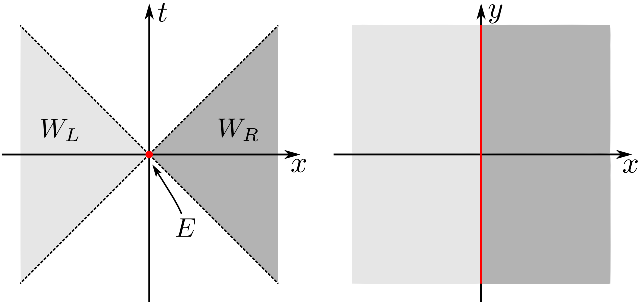

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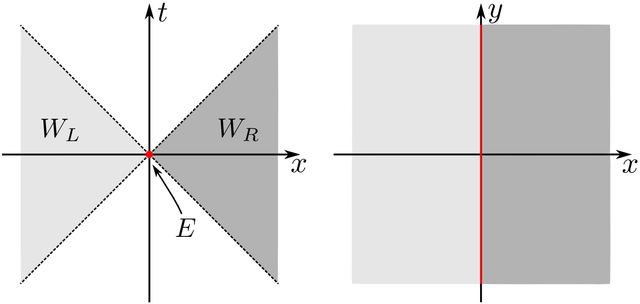

in the vacuum representation

- edge $E =$ orbit of $\{0\}$ under the flow of the Killing vector fields (∂_y, ∂_z)

$$W_R \cup W_L = E', \quad W_R \cap W_L = \varnothing, \quad {W_R}' = W_L$$

Warped convolution on Minkowski spacetime

- **Warped operators**: Let $A \in \mathcal{A}(W_R)$ be 'smooth' w.r.t. translations.

$$A_\lambda := \frac{1}{4\pi^2} \int_{\mathbb{R}^2 \times \mathbb{R}^2} dx\, dy\, e^{-ixy}\, U(\lambda Q x) A U(-\lambda Q x) U(y), \quad x, y \in E,$$

where $Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\lambda \in \mathbb{R}$ and U is a continuous representation of translations along the edge = isometries generated by (∂_y, ∂_z)

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- Let $\mathcal{A}(W_R)_\lambda := \{A_\lambda : A \in \mathcal{A}(W_R) \text{ is smooth}\}''$. The map

$$W := (a, \Lambda) W_R \longmapsto U(a, \Lambda) \mathcal{A}(W_R)_\lambda U(a, \Lambda)^{-1} =: \mathcal{A}(W)_\lambda$$

is a wedge local net and describes non-trivial interaction

$${}^{\text{out}}_{\lambda}\langle p, q | p', q' \rangle^{\text{in}}_{\lambda} = e^{i|\lambda p Q q| + i|\lambda p' Q q'|} \ . \ {}^{\text{out}}\langle p, q | p', q' \rangle^{\text{in}}$$

Going to curved spacetimes

Goal: Warped convolution for QFTs on (a class of) curved spacetimes.

Basic questions:

- What is a wedge in a curved spacetime?

- What is the $\mathbb{R}^2$-action for the deformation?

- How to generate the defomed net?

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  - (anti) de Sitter: [Buchholz,Borcher:99] [Buchholz,Summers:04] [Rehren:00]
  - asymptotically anti de Sitter: [Lauridsen-Ribeiro:07]
  - FRW with spherical spatial sections: [Buchholz,Mund,Summers:01]
  - Schwarzschild: [Guido,Longo,Roberts,Verch:01] [Kay:85]
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- What is the $\mathbb{R}^2$-action for the deformation?
  The Killing flow.
- How to generate the defomed net?
  Use a family of algebras/wedges.

Admissible spacetimes

A globally hyperbolic spacetime $(M \cong \mathbb{R} \times \Sigma, g)$ is called **admissible**, if

- it admits two spacelike Killing vector fields ξ_1, ξ_2 which are smooth, complete, commuting and linearly independent at each point
- M splits according to $M \cong \mathbb{R} \times I \times E$ where I is simply connected and
$$E = \{\varphi_{\xi_1,s_1}(\varphi_{\xi_1,s_2}(p)) : s_1, s_2 \in \mathbb{R}\}$$
- there exists a smooth embedding $\iota : \mathbb{R} \times I \to M$.

Hence, $(\mathbb{R} \times I, \iota^* g)$ is globally hyperbolic without null focal points.

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Killing pairs $\Xi(M, g) =$ the set of all *ordered* pairs (ξ_1, ξ_2) satisfying these conditions

Inverted Killing pair: $\xi' = (\xi_2, \xi_1)$

Examples of admissible spacetimes

- **certain warped product spacetimes**
- **Bianchi models I-VII**

- **Kasner spacetimes**
  - spatially homogeneous but anisotropic cosmological models
  - $M \cong J \times \mathbb{R}^3$ with $J \subset \mathbb{R}$ and

$$g = dt^2 - e^{2f_1} dx^2 - e^{2f_2} dy^2 - e^{2f_3} dz^2,$$

  - $\mathrm{Iso}(M, g) \supset \mathbb{R}^3$.

- **FRW spacetimes with flat spatial sections**
  - spatially homogeneous and isotropic cosmological models
  - $M \cong J \times \mathbb{R}^3$ with $J \subset \mathbb{R}$ and

$$g = dt^2 - a^2(t) \Big[dx^2 + dy^2 + dz^2 \Big],$$

  - $\mathrm{Iso}(M, g) \supset \mathbb{R}^3 \rtimes O(3)$.

- **Minkowski spacetime**

Properties of Killing Pairs

Actions on Killing pairs:

- $\mathrm{Iso}(M, g)$ acts on $\Xi(M, g)$ via push-forward: $h_*\xi := (h_*\xi_1, h_*\xi_2)$
- $\mathrm{GL}(2, \mathbb{R})$ acts on $\Xi(M, g)$ pointwise: $(N\xi)(p) := N(\xi_1(p), \xi_2(p))$

These are commuting group actions.

Associated flow:

- $\varphi_{\xi,s} := \varphi_{\xi_1,s_1} \circ \varphi_{\xi_2,s_2}$, where $s := (s_1, s_2) \in \mathbb{R}^2$
- φ_ξ is an isometric $\mathbb{R}^2$-action by diffeomorphisms on (M, g).

Edges and wedges in admissible spacetimes

Definition: An **edge** is a subset of M which has the form

$$E_{\xi,p} := \{\varphi_{\xi,s}(p) \in M : s \in \mathbb{R}^2\}$$

for some $\xi \in \Xi(M, g)$ and $p \in M$.

Remarks:

- An edge is a two-dimensional, spacelike, smooth submanifold of M.
- An edge can have the topology of a plane, cylinder or torus.

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Lemma

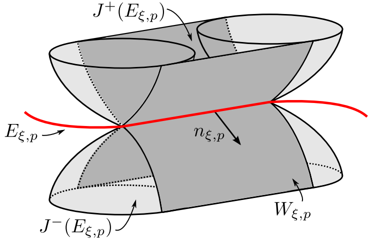
The causal complement $E'_{\xi,p}$ of an edge $E_{\xi,p}$ is the disjoint union of two connected components, which are causal complements of each other.

This Lemma is not true if the topological restriction on M is dropped.

Edges and wedges in admissible spacetimes

<u>Definition:</u> • A **wedge** is a connected component of the causal complement of an edge.

- Distinguish the two components, $W_{\xi,p}$ and $W_{\xi',p}$, via orientation.
- Family of wedges: $\mathcal{W} := \{W_{\xi,p} : \xi \in \Xi(M,g), p \in M\}$



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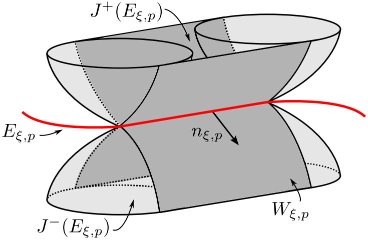

Proposition (Properties of wedges)

- $(W_{\xi,p})'' = W_{\xi,p}$. Hence $W_{\xi,p}$ is globally hyperbolic.
- $(W_{\xi,p})' = W_{\xi',p}$
- $\varphi_{\xi,s}(W_{\xi,p}) = W_{\xi,p}$, for all $s \in \mathbb{R}^2$
- $h(W_{\xi,p}) = W_{h_*\xi,h(p)}$

Hence, $\mathcal{W}$ is invariant under $\mathrm{Iso}(M,g)$ and causal complementation.

Edges and wedges in curved spacetimes

Divide $\mathcal{W}$ into **coherent subfamilies**:

- Equivalence relation: $\xi, \tilde{\xi} \in \Xi(M, g)$

$$\xi \sim \tilde{\xi} :\Leftrightarrow \exists h \in \mathrm{Iso}(M, g) \text{ and } N \in \mathrm{GL}(2, \mathbb{R}) : \ \tilde{\xi} = N h_* \xi$$

- $\mathcal{W} = \bigsqcup_{[\xi]} \mathcal{W}_{[\xi]}, \quad \mathcal{W}_{[\xi]} = \{W_{\tilde{\xi},p} : \tilde{\xi} \sim \xi,\, p \in M\}$. We have:

$$\mathcal{W}_{[\xi]} = \{W_{h_*\xi,p}, W'_{h_*\xi,p} : h \in \mathrm{Iso}(M, g),\, p \in M\}.$$

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Each $\mathcal{W}_{[\xi]}$ is invariant under $\mathrm{Iso}(M,g)$ and causal complementation.

Proposition (Inclusion of wedges)

$W_{\xi,p} \subset W_{\tilde{\xi},\tilde{p}} :\Leftrightarrow \ p \in \overline{W_{\tilde{\xi},\tilde{p}}}$ and $\exists N \in \mathrm{GL}(2,\mathbb{R}), \, \det N > 0 \ : \tilde{\xi} = N\xi$.

Hence, only wedges in the same coherent subfamily can form an inclusion.

Deformation of wedge-local nets with Killing vector fields

Let $W \mapsto \mathcal{A}(W) \subset \mathcal{A}$ be a wedge-local net on an admissible spacetime:

(1) $W \subset \tilde{W} \;\Rightarrow\; \mathcal{A}(W) \subset \mathcal{A}(\tilde{W})$

(2) $W \subset \tilde{W}' \Rightarrow\; \mathcal{A}(W) \subset \mathcal{A}(\tilde{W})'$

(3) There exists a strongly continuous action $\alpha : \mathrm{Iso}(M,g) \to \mathrm{Aut}(\mathcal{A})$:

$$\alpha_h(\mathcal{A}(W)) = \mathcal{A}(hW), \quad h \in \mathrm{Iso}(M,g).$$

<u>Goal:</u> use warped convolution to define a map $W \mapsto \mathcal{A}(W)_\lambda$ with $\lambda \in \mathbb{R}$:

- $\mathcal{A}(W)_0 = \mathcal{A}(W)$
- (1), (2), (3) is true for $W \mapsto \mathcal{A}(W)_\lambda$ and each $\lambda \in \mathbb{R}$.

Warped convolution with Killing vector fields

For the deformation: work in a Hilbert space setting

- $\mathcal{A} \subset \mathcal{B}(\mathcal{H})$
- there exists a continuous unitary representation U of $\mathrm{Iso}(M, g)$:

$$\alpha_g(A) = U(g)AU(g)^{-1}, \quad A \in \mathcal{A}.$$

Remarks:

- every C^*-dynamical system $(\mathcal{A} \subset \mathcal{B}(\mathcal{H}), G, \alpha)$ has a faithful and covariant representation on $L^2(G) \otimes \mathcal{H}$ [Pedersen:78]
- assume that $\mathcal{H}$ is separable
- no state is selected
  (no assumption about about U-invariant vectors or spectral properties of U)

Warped convolution with Killing vector fields

Ingredients for the deformation:

- strongly continuous, automorphic $\mathbb{R}^2$-action:

$$\alpha_{\varphi_{\xi,s}} = \mathrm{ad}\, U(\varphi_{\xi,s}), \quad \tau_{\xi,s} := \alpha_{\varphi_{\xi,s}}, \quad U_\xi(s) := U(\varphi_{\xi,s})$$

- smooth elements w.r.t. $\xi \in \Xi(M,g)$:

$$\mathcal{A}^\infty := \{A \in \mathcal{A} : \mathbb{R}^2 \ni s \mapsto \tau_{\xi,s}(A) \text{ is smooth in norm}\}$$

- $Q = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Definition: Let $A \in \mathcal{A}^\infty$ and define (in an oscillatory sense)

$$A_{\xi,\lambda} := \frac{1}{4\pi^2} \int ds\, ds'\; e^{-iss'} U_\xi(\lambda Q s) A U_\xi(-\lambda Q s) U_\xi(s'),$$

where $\lambda \in \mathbb{R}$ and ss' is the standard Euclidean inner product of $s, s' \in \mathbb{R}^2$.

Warped convolution with Killing vector fields

Properties of warped operators: [Buchholz,Lechner,Summers:10]

- $A_{\xi,\lambda}$ is well-defined on a dense subspace of $\mathcal{H}$ and can be extended to bounded operator on all of $\mathcal{H}$
- $A_{\xi,0} = A$
- $(A_{\xi,\lambda})^* = (A^*)_{\xi,\lambda}$
- $A_{\xi,\lambda} B_{\xi,\lambda} = (A \times_{\xi,\lambda} B)_{\xi,\lambda}$, where $\times_{\xi,\lambda}$ is the Rieffel product

$$A \times_{\xi,\lambda} B := \frac{1}{4\pi^2} \int ds\, ds'\, e^{-iss'} \tau_{\xi,\lambda Q s}(A) \tau_{\xi,s'}(B)$$

- If $[\tau_{\xi,s}(A), B] = 0$ for all $s \in \mathbb{R}^2$, then $[A_{\xi,\lambda}, B_{\xi,-\lambda}] = 0$.

Deformation of wedge-local nets

Minkowski: Deforming a wedge-local net = deforming a single algebra.

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Procedure:

- Consider $\mathcal{W}_{[\xi]} \subset \mathcal{W}$ and choose a representative $\xi \in [\xi]$
- for $W_{\xi,p} \in \mathcal{W}_{[\xi]}$ define

$$\mathcal{A}(W_{\xi,p})_\lambda := \{A_{\xi,\lambda} : A \in \mathcal{A}(W_{\xi,p})^\infty\}^{\|\cdot\|}$$
$$\mathcal{A}(W'_{\xi,p})_\lambda := \{A_{\xi,-\lambda} : A \in \mathcal{A}(W'_{\xi,p})^\infty\}^{\|\cdot\|}$$

- extend this to arbitrary wedges $hW_{\xi,p} \in \mathcal{W}_{[\xi]}$:

$$\mathcal{A}(hW_{\xi,p})_\lambda := \alpha_h(\mathcal{A}(W_{\xi,p})_\lambda), \quad \mathcal{A}(hW'_{\xi,p})_\lambda := \alpha_h(\mathcal{A}(W'_{\xi,p})_\lambda).$$

This defines

$$W \longmapsto \mathcal{A}(W)_\lambda$$

for all $W \in \mathcal{W}$, as $[\xi]$, p and h vary over $\Xi(M,g)/\sim$, M and $\mathrm{Iso}(M,g)$.

Properties of the deformed net

Theorem

The map $W \mapsto \mathcal{A}(W)_\lambda$ is a wedge-local net, i.e.

- $W \subset \tilde{W} \;\Rightarrow\; \mathcal{A}(W)_\lambda \subset \mathcal{A}(\tilde{W})_\lambda$
- $W \subset \tilde{W}' \;\Rightarrow\; \mathcal{A}(W)_\lambda \subset \mathcal{A}(\tilde{W})_\lambda{}'$, i.e.

$$[A_{\xi,\lambda}, B_{\xi,-\lambda}] = 0, \quad A \in \mathcal{A}(W_{\xi,p}),\; B \in \mathcal{A}(W'_{\xi,p})$$

- $\alpha_h(\mathcal{A}(W)_\lambda) = \mathcal{A}(hW)_\lambda, \quad h \in \mathrm{Iso}(M,g)$.
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Remark:

- The deformation depends on the choice of representatives ξ.
  ($\equiv$ choice of length scale of the Killing flows)
- $(\xi \to c\cdot\xi,\, c \in \mathbb{R}) \Rightarrow (\varphi_{\xi,s} \to \varphi_{c\xi,s} = \varphi_{\xi,cs}) \Rightarrow (\lambda \to c^2\lambda)$

Properties of the deformed net

- $W \mapsto \mathcal{A}(W)_\lambda$ can be interpreted as an effective theory on a noncommutative spacetime [Grosse,Lechner:07,08]

  It is expected that $\bigcap_{W \supset \mathcal{O}} \mathcal{A}(W)_\lambda = \mathbb{C} \cdot 1$ for $\mathcal{O}$ bounded and $\lambda \neq 0$ (no proof yet).

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- **Concrete model:** free Dirac field on FRW with flat spatial sections
  - consider $\mathrm{E}(3)$-invariant, quasifree state ω with Reeh-Schlieder property (spacetime deformation argument [Sanders:08])
  - GNS representation of the Dirac field $\rightarrow$ warped convolution
  - Reeh-Schlieder is violated for regions $\mathcal{C}$ smaller than wedges
  - Hence, there does not exist a unitary V, such that

$$V \pi_\omega(\mathcal{A}(\mathcal{C}))_\lambda V^{-1} = \pi_\omega(\mathcal{A}(\mathcal{C})),$$

  since V would preserve cyclicity of Ω_ω.

Conclusion

We have defined wedges in a class of spacetimes which admit two spacelike Killing VF.

- These share the essential features of wedges in Minkowski spacetime.
- The family $\mathcal{W}$ is, in general, *not* causally seperating:
  given double cones $\mathcal{O}_1, \mathcal{O}_2 \subset M$ spacelike separated
  $\Rightarrow \exists W \in \mathcal{W} : \ \mathcal{O}_1 \subset W \subset \mathcal{O}_2'$
  This property is important to go back to local nets.
  (However, in the FRW-case, $\mathcal{W}$ *is* causally seperating.)

We have applied the warped convolution deformation method to QFTs on these spacetimes.

- The Killing flow was used as $\mathbb{R}^2$-action.
- Localization of observables in wedges is stable under deformation.

Outlook

Applications in cosmology? (4-point function)

Non-admissible spacetimes:

- FRW with spherical spatial sections ($E \cong S^2$)
- (4-d) de Sitter
  - $E \cong S^2 = \mathrm{SO}(3)$-orbit
  - abelian Killing flows
- Schwarzschild
- Kerr

Deformations with non-abelian groups

- Poincaré group
- affine group [Bieliavsky:07] $\longrightarrow$ conformal QFTs