

EU-NCG: Focused Semester on Mathematical Physics and NCG

School on Planar Algebras and Physics

15 February – 5 March 2010

School of Mathematics, Cardiff University

Speakers:

- Julien Bichon (Clermont-Ferrand): *Quantum permutation groups*
1. Compact quantum groups (algebraic approach)
 2. The quantum permutation group and its representation category
 3. Quantum symmetries of finite graphs
- Stephen Bigelow (UCSB): Presentations of planar algebras in terms of generators and relations
- Richard Burstein (Vanderbilt) *Bipartite graph planar algebras*
1. Introduction to the bipartite graph planar algebra
 2. Bipartite graph planar algebras and subfactors
 3. Applications to subfactors: planar fixed-point subfactors
 4. Applications to subfactors: Hadamard subfactors
- Jürgen Fuchs (Karlstad): *CFT, TFT, algebras in tensor categories, and all that*
1. The world sheet picture of CFT
 2. Ribbon categories and graphical calculus
 3. Algebras in tensor categories
 4. TFT and link invariants
 5. The TFT construction of RCFT correlators
- Shamindra Ghosh (KU Leuven) Planar algebras and their representations.
- Pinhas Grossman (Cardiff) Planar primer
- Marcos Mariño (Geneva): Quantum theory and enumerative problems
- Richard Thomas (Imperial): Curves on K3 surfaces and modular forms
- Ben Davison (Oxford): What are brane tiling algebras, and when are dimer model algebras Calabi-Yau?
- Ved Gupta (Leuven): Affine morphisms at zero level
- Amihay Hanany (Imperial): On the classification of brane tilings

Yang-Hui He (Oxford):	Quivers, gauge theories and Calabi-Yau singularities
Pawel Kasprzak (Copenhagen):	Rieffel deformation of homogeneous spaces
Alastair King (Bath):	Dimers and Calabi-Yau-3 algebras
Eric Morfa-Morales (ESI, Vienna):	Rieffel deformations of quantum field theories
Mathew Pugh (Cardiff):	A_2 -planar algebras
Alex Quintero-Velez (Glasgow):	A-infinity structures associated with brane tilings and D-brane categories
Sven Raum (Leuven):	Fusion rules of free orthogonal quantum groups
Peter West (King's):	E Theory

Timetable of lectures:**Week 1**

15-19 Feb	Jürgen Fuchs (Karlstad) Pinhas Grossman (Cardiff)
15 Feb	Alastair King (Bath)
16 Feb	Ben Davison (Oxford)
18 Feb	Yang-Hui He (Oxford)
19 Feb	Sven Raum (Leuven)

Week 2

22-25 Feb	Richard Burstein (Vanderbilt) Shamindra Ghosh (KU Leuven)
22&25 Feb	Mathew Pugh (Cardiff)
23 Feb	Alex Quintero-Velez (Glasgow) Ved Gupta (Leuven)
24 Feb	Peter West (King's)
26 Feb	Marcos Mariño (Geneva) Richard Thomas (Imperial)

Week 3

1-5 March	Stephen Bigelow (UCSB)
1 March	Pinhas Grossman (Cardiff)
2-4 March	Julien Bichon (Clermont)
3 March	Pawel Kasprzak (Copenhagen)
5 March	Eric Morfa-Morales (ESI, Vienna) Amihay Hanany (Imperial)

Abstracts:

Julien Bichon (Clermont-Ferrand)

Quantum permutation groups

1. *Compact quantum groups (algebraic approach)*
2. *The quantum permutation group and its representation category*
3. *Quantum symmetries of finite graphs*

Abstract: The aim of this series of talks is to provide an introduction to Wang's quantum permutation group. We shall describe its representation category by using the category of non-crossing partitions (a combinatorial version of Temperley-Lieb type categories), a recent approach due to Banica-Speicher-Vergnioux. If time permits, we shall also discuss some quantum subgroups of the quantum permutation group, such as those associated to finite graphs.

Stephen Bigelow (UCSB)

Presentations of planar algebras in terms of generators and relations

Abstract: This series of talks takes a concrete algorithmic approach to planar algebras, with a focus on subfactor planar algebras of low index. I plan to cover the following topics:

The principal graph of a subfactor planar algebra: what it means, and how to extract information from it, including the existence of generators satisfying certain relations. The "jellyfish algorithm", which uses certain relations to put any diagram into a standard form. By a dimension count, we can then show that we have enough relations to give a presentation of the planar algebra.

Reversing the process: if we did not already know the existence of a planar algebra with a given principal graph, could we prove it directly from the presentation? This is how, in joint work with Morrison, Peters and Snyder, we proved the existence of the "extended Haagerup" planar algebra.

Richard Burstein (Vanderbilt)***Bipartite graph planar algebras****1. Introduction to the bipartite graph planar algebra*

Abstract: this will cover the basic properties of BGPAs, including Vaughan's original definition and the extension to infinite graphs

2. Bipartite graph planar algebras and subfactors

Abstract: this will cover the reasons why BGPAs are relevant to the study of subfactors: small planar subalgebras of BGPAs are of subfactor type.

3. Applications to subfactors: planar fixed-point subfactors

Abstract: this will cover my result on automorphisms of BGPAs, with applications to subfactors

4. Applications to subfactors: Hadamard subfactors

Abstract: this will cover my older result on Hadamard matrices and subfactors, using some more recent results to place this in the context of BGPAs

Jürgen Fuchs (Karlstad)***CFT, TFT, algebras in tensor categories, and all that***

- 1. The world sheet picture of CFT*
- 2. Ribbon categories and graphical calculus*
- 3. Algebras in tensor categories*
- 4. TFT and link invariants*
- 5. The TFT construction of RCFT correlators*

Abstract: The "TFT construction" allows one to compute the correlators of rational conformal field theories, on arbitrary world sheets and with any number of field insertions, by expressing them as invariants of ribbon graphs in three-manifolds. Before describing this construction, I will introduce essential ingredients and tools, such as various structures on the world sheet (boundary conditions, defect lines, field insertions), ribbon categories and a graphical calculus for working with them, modular tensor categories, Frobenius algebras in tensor categories, and three-dimensional topological field theory.

References:

- B. Bakalov and A.A. Kirillov, Lectures on Tensor Categories and Modular Functors (AMS 2001)
- J. Fuchs, I. Runkel, C. Schweigert, hep-th/0204148
- J. Fuchs, I. Runkel, C. Schweigert, hep-th/0412290
- J. Fjelstad, J. Fuchs, I. Runkel, C. Schweigert, hep-th/0503194, appendices A and B

Shamindra Ghosh (KU Leuven)*Planar algebras and their representations.*

Abstract: I will recall the definition of Jones's planar algebra and discuss an important relation, namely, the 'higher exchange relation' (which is a generalization of 'exchange relation' obtained by Bisch in the context of intermediate subfactors and by Landau for planar algebras and appears in many interesting planar algebras) acting as a sufficient condition for finite-dimensionality. I will then construct a planar algebra from a 'pivotal bicategory'. Starting with a 'subfactor planar algebra', I will describe the construction (given by Jones-Shlyakhtenko-Walker/Kodiyalam-Sunder) of a subfactor with the same planar algebra associated to it.

Pinhas Grossman (Cardiff)*Planar primer*

Abstract: These talks will be an introduction to Jones' theory of planar algebras. I will define planar algebras, explain their relation to subfactors, and describe some examples. I will also discuss the application of planar algebra techniques to the construction and classification of subfactors as well as some problems and challenges that arise, and finally present an overview of current progress in this area.

Marcos Mariño (Geneva)*Quantum theory and enumerative problems*

Abstract: Many problems in mathematics involve counting objects. For example, in combinatorics, one is often interested in counting graphs with certain conditions. Another example is enumerative algebraic geometry, where one counts curves in algebraic varieties. Many of these problems turn out to be deeply related to quantum theories. In this talk I will explore this relation in three important examples: the enumeration of maps, the enumeration of Hurwitz coverings, and the LMO invariant of rational homology spheres. I will show how ideas of quantum theory lead to new insights in these problems, and in particular I will also explain the connection between non-perturbative effects in quantum theory and the asymptotic behavior of these enumerative problems.

Richard Thomas (Imperial)*Curves on K3 surfaces and modular forms*

Abstract: The Katz-Klemm-Vafa formula is a conjecture expressing Gromov-Witten invariants of K3 surfaces in terms of modular forms. In genus 0 it reduces to the (proved) Yau-Zaslow formula. I will explain how a correspondence between "stable pairs" and Gromov-Witten theory for toric 3-folds (proved by Maulik-Oblomkov-Okounkov-Pandharipande), some calculations with stable pairs (due to Kawai-Yoshioka) and some deformation theory lead to a proof of the KKV formula.

Ben Davison (Oxford)*What are brane tiling algebras, and when are dimer model algebras Calabi-Yau?*

Abstract: Brane tiling algebras are a type of superpotential algebra, an import from Physics. In this talk I will define what they are, and discuss the conditions under which they are Calabi-Yau, as well as briefly discussing a sense in which they are "always" Calabi Yau.

Ved Gupta (Leuven)*Affine morphisms at zero level*

Abstract: We show that the affine morphisms at zero level in the affine category over the planar algebra of a finite depth extremal subfactor $N \subset M$ is isomorphic to the fusion algebra of equivalence classes of N - N , N - M , M - N , M - M bimodules appearing in the standard invariant of the subfactor. This is a joint work with Paramita Das and Shamindra Ghosh.

Amihay Hanany (Imperial)*On the classification of brane tilings*

Abstract: Brane tilings give a large class of SCFT's in 3+1 and 2+1 dimensions. In this talk I will discuss several attempts to classify all such models. Statistical properties of these models can be derived using some techniques in crystallography and in number theory.

Yang-Hui He (Oxford)

Quivers, gauge theories and Calabi-Yau singularities

Abstract: We discuss some recent progress on how quiver theories inevitably arise within string theory as world-volume theory of branes in various spacetime dimensions. We discuss intimate relations between the gauge theory, encoded by the quiver representations, the algebraic geometry of affine Calabi-Yau varieties, and auxiliary combinatorics involving dimers, tilings and plethystics.

Pawel Kasprzak (Copenhagen)

Rieffel deformation of homogeneous spaces

Abstract: Let H be a closed subgroup of a locally compact group G , and let X be the quotient space of cosets. Also, let L be an abelian closed subgroup of H and f be a 2-cocycle on the dual group of L . Using these data, we can define an action of the quantum group $G(f)$ (the Rieffel deformation of G) on the Rieffel deformation of X . On the other hand, we can perform the Rieffel deformation of the subgroup H obtaining the quantum subgroup $H(f)$ of $G(f)$. This in turn, by the results of Vaes, leads to the C^* -algebraic quotient $G(f)/H(f)$. The aim of this talk is to show that two constructions described above give isomorphic quantum homogeneous spaces.

Alastair King (Bath)

Dimers and Calabi-Yau-3 algebras

Abstract: I will explain how the Calabi-Yau-3 condition transfers from geometry to non-commutative algebra and how a novel use of dimer models in string theory leads to many examples.

Eric Morfa-Morales (ESI, Vienna)

Rieffel deformations of quantum field theories

Abstract: I will explain how Rieffel's deformation procedure of C^* -algebras can be applied to nets of observable algebras in algebraic quantum field theory. The deformed net exhibits a weakened form of locality and a non-trivial scattering matrix, i.e. it describes an interacting quantum field theory. I will show how this novel constructive approach can be applied to vacuum theories on Minkowski spacetime and to generic theories on a certain class of cosmological spacetimes.

Mathew Pugh (Cardiff)*A₂-planar algebras*

Abstract: An A₂-planar algebra is a generalization of Jones's notion of a planar algebra which captures the structure contained in the SU(3) ADE subfactors which rely on the existence of a cell system which defines a connection or Boltzmann weight. I will show that the subfactor for a finite ADE graph with a flat cell system has a description as a flat A₂-planar algebra. A description of the (Jones) planar algebra for the Wenzl subfactor in terms of generators and relations is obtained using A₂-planar algebras. This work also provides a framework for studying the subfactors for other SU(3) ADE graphs, and subfactors for other graphs which carry cell systems, both those which have index less than 9 and those in the continuous regime in SU(3).

Alex Quintero-Velez (Glasgow)*A-infinity structures associated with brane tilings and D-brane categories*

Abstract: Let Z be a weak Fano toric surface and denote by $X = \text{tot}(K_Z)$ the total space of its canonical line bundle. As argued by Hanany et al., there is a consistent brane tiling (aka dimer model) corresponding to the toric diagram of Z . In this talk, I construct the natural A-infinity structure in the associated D-brane category $D_Z(\text{Coh}(X))$. Then I show that it precisely reproduces the results of the method of brane tiling.

Sven Raum (Leuven)*Fusion rules of free orthogonal quantum groups*

Abstract: Free orthogonal quantum groups are a class of sub-quantum groups of the free orthogonal Wang algebra which can be described by combinatorial means. Actually Banica and Speicher were able to classify them and it turns out that there are only six such algebras. As these quantum groups are more easily accessible than other ones it is natural to ask questions about their structure. We continued work of Banica, Vergnioux and others by calculating the fusion rules of all not yet considered free orthogonal quantum groups and of their free complexifications. We can conclude that not all free orthogonal quantum groups admit a free fusion ring. This statement will be explained and related to other observations concerning free orthogonal quantum groups.

Peter West (King's)*E Theory*

Abstract: I will review the current status of string theory and explain the evidence for a very large Kac-Moody symmetry of an underlying theory of strings and branes.