## EU-NCG: Focused Semester on Mathematical Physics and NCG

# EU-NCG Third Annual Meeting

*28 June – 2 July 2010*

School of Mathematics, Cardiff University

**Speakers:**

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| Iakovos Androulidakis (Göttingen): | *Pseudodifferential calculus for singular foliations* |
| Dorothea Bahns (Göttingen): | *Quasiplanar Wick products* |
| Tomasz Brzezinski (Swansea): | *Noncommutative integral forms* |
| Kenny De Commer (Rome): | *Galois co-objects for von Neumann algebraic quantum groups* |
| Bergfinnur Durhuus (Copenhagen): | *The scattering problem for a noncommutative nonlinear Schrödinger equation* |
| Pinhas Grossman (Cardiff): | *A quadrilateral in the Asaeda-Haagerup category* |
| Daniele Guido (Rome): | *Some (noncommutative) geometrical aspects of the Sierpinski gasket* |
| Emmanuel Germain (Caen): | *Focused Semester 7: Quantum Groups* |
| Robin Hillier (Rome): | *Spectral Triples and an Index Pairing for Super-Conformal Nets* |
| Stefan Hollands (Cardiff): | *Perturbed vertex algebras and perturbative quantum field theories* |
| Tommaso Isola (Rome): | *Line integrals of one-forms on the Sierpinski gasket* |
| Pawel Kasprzak (Copenhagen): | *Quantum homogeneous spaces* |
| Yasuyuki Kawahigashi (Tokyo): | *Superconformal field theory and operator algebras* |
| Claus Köstler (Aberystwyth): | *Symmetries and invariance principles in noncommutative probability* |
| Amin Malik (Oslo): | *On a quantization formula for a class of symplectic symmetric space* |

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| Jouko Mickelsson (Helsinki): | *Families of Dirac operators and quantum affine algebras* |
| Eric Morfa-Morales (ESI, Vienna): | *Deformations of quantum field theories on spacetimes with Killing vector fields* |
| Radu Munteanu (Bucharest): | *Fixed point factors under product type actions on ITPFI factors* |
| Denjoe O'Connor (DIAS): | *TBA* |
| Eduard Ortega (Copenhagen): | *The Cuntz semigroup and the open projection* |
| Liviu Paunescu (Rome): | *Sofic equivalence relations and Connes' embedding problem* |
| Mihai Popa (IMAR): | *Some applications of non-commutative functions in operator-valued non-commutative probability* |
| Thomas Schucker (Provence): | *Noncommutative geometry and the standard model* |
| Wojciech Szymanski (Odense): | *Endomorphisms of Cuntz algebras* |
| Hannes Thiel (Copenhagen): | *The Chern character for low-dimensional spaces and applications* |
| Otgonbayar Uuye (Copenhagen): | *Homotopical algebra for C*-algebras* |
| André Verbeure (K.U. Leuven): | *Spontaneous Symmetry Breaking* |
| Dan Voiculescu (UCB): | *Probability and analysis at the highest degree of noncommutativity* |
| Mihaly Weiner (Rome): | *An algebraic Haag's theorem* |
| Bora Yalkinoglu (Paris VI): | *On Bost-Connes type systems and Complex Multiplication* |
| Jakob Yngvason (ESI, Vienna): | *Quantum fields localized in cones and wedges* |
| Laszlo Zsido (Rome): | *Relative weakly mixing for W*-dynamical systems* |

# Abstracts:

**Iakovos Androulidakis (Göttingen)**

*Pseudodifferential calculus for singular foliations*

Abstract: The holonomy groupoid of a singular foliation $(M,F)$ is a very ill-behaved object, nevertheless in earlier work we showed that one can construct the associated $C^*$-algebra. In this lecture, after a brief overview of this construction, we discuss how the multipliers of this $C^*$-algebra allow for the definition of longitudinal pseudodifferential operators. An application is the construction of the Laplacian of a foliation $F$ on a compact manifold $M$ as an unbounded, self-adjoint operator of $L^2(M)$. We also discuss the analytic index and how it can be recovered from a tangent groupoid (in the sense of A. Connes) naturally associated with the foliation. This is joint work with G. Skandalis.

**Dorothea Bahns (Göttingen)**

*Quasiplanar Wick products*

Abstract: In quantum field theoretic models on vector spaces, the class of admissible interaction terms is essentially fixed by requirements such as Poincare invariance and locality. On noncommutative spaces, these requirements have to be weakened and hence, there is some freedom in the choice possible interaction terms. I will recall two different such generalizations: one which is based on twisting the tensor product of distributions and a weak notion of locality, the other which is based on redefining the limit of coinciding points in an algebraic setting. Concentrating on the former approach in this talk, I will discuss a notion of generalized Wick products in this framework ("quasiplanar Wick products") and report on recent progress in understanding the underlying functional analysis. This is joint work with Sergio Doplicher, Klaus Fredenhagen, and Gherardo Piacitelli.

**Tomasz Brzezinski (Swansea)**

*Noncommutative integral forms*

Abstract: The notion of a complex of integral forms on a non-commutative space is described. Following Manin, this is defined as a complex with a boundary operator given by a flat hom-connection (or a flat right connection in the terminology of Manin). We show how hom-connections, and therefore complexes of integral forms, can be constructed from free twisted multi-derivations. In particular, we show that a Hopf algebra with a left covariant differential calculus admits a hom-connection. Examples of integral forms include such forms on the matrix algebra (with the Lie algebra or derivation based calculus), on the quantum group SU$q$(2) (with the 3D calculus), and on the standard Podles sphere (with the calculus originated from the 3D calculus). In each case the complex of integral forms is shown to be isomorphic to the corresponding non-commutative de Rham complex, thus reflecting perfectly the classical case and also pointing in the direction of the Poincare duality. The constructed integrals coincide with the trace based integral of Dubois-Violette, Kerner and Madore (the matrix algebra case) and with the Haar integral (in the quantum group case). Based on joint work with Laiachi El Kaoutit and Christian Lomp.

**Kenny De Commer (Rome)**

*Galois co-objects for von Neumann algebraic quantum groups*

Abstract: We will explain the concept of a Galois co-object for a von Neumann algebraic quantum group (vN aqg), which generalizes the notion of a unitary 2-cocycle. Also the procedure of `twisting with a 2-cocycle' can be generalized to this setting. As an application, we show that for the group von Neumann algebra of a torsionless discrete group, considered as a vN aqg, any Galois co-object is trivial. In particular, such a vN aqg does not allow for any interesting twisting (or deformation).

**Bergfinnur Durhuus (Copenhagen)**

*The scattering problem for a noncommutative nonlinear Schrödinger equation*

Abstract: We investigate the Cauchy problem and scattering properties for a Moyal deformed version of the nonlinear Schrödinger equation in an even number of space dimensions. With rather weak conditions on the degree of nonlinearity, the Cauchy problem for general initial data has a unique globally defined solution, and also has solitary wave solutions if the interaction potential is appropriately chosen. We demonstrate how to set up a scattering framework in certain operator spaces for equations of this type, including decay estimates of the free time evolution and the construction of wave operators defined for small scattering data in the general case and for arbitrary scattering data in the rotationally symmetric case. This is joint work with Victor Gayral.

**Pinhas Grossman (Cardiff)**

*A quadrilateral in the Asaeda-Haagerup category*

Abstract: We will describe the construction of a quadrilateral of factors whose upper sides are each the Asaeda-Haagerup subfactor, using Longo's notion of a Q-system, Ocneanu's paragroup theory, and the diagrammatic calculus for tensor categories. This is joint work with M. Asaeda.

**Daniele Guido (Rome)**

*Some (noncommutative) geometrical aspects of the Sierpinski gasket*

Abstract: We present here a 2-parameter family of spectral triples for the Sierpinski gasket, based on spectral triples for the circle. Any hole (lacuna) of the gasket is suitably identified with a circle, and the triple for the gasket is defined as the direct sum of the triples for the lacunas. The first parameter is a scaling parameter for the correspondence between circles and lacunas, the second describes the metric on the circle, which is, roughly, a power of the Euclidean metric. We study for which parameters the following features of the gasket can be recovered by the corresponding triple: the integration on the gasket (w.r.t. the Hausdorff measure), a non-trivial distance on the gasket, a non-trivial Dirichlet form (the Kigami energy). Joint work with F. Cipriani, T. Isola and J-L. Sauvageot.

**Emmanuel Germain (Caen)**

*Focused Semester 7: Quantum Groups*

http://www.math.unicaen.fr/~aps/qsem/index.html

**Robin Hillier (Rome)**

*Spectral Triples and an Index Pairing for Super-Conformal Nets*

Abstract: During the past decades, noncommutative geometry (in particular spectral triples) turned out to be very successful in describing several aspects of mathematics and physics. One might wonder whether such a description also makes sense in the case of (super-)conformal quantum field theory? What would we like to have? How can we build up a connection between the two? Do we get new insights or even new invariants for conformal nets? We study these questions and a solution in the example of super-current algebra models, with an outlook on further models. Finally, we generalise these ideas to arbitrary super-conformal nets -- as far as possible.

**Stefan Hollands (Cardiff)**

*Perturbed vertex algebras and perturbative quantum field theories*

Abstract:

**Tommaso Isola (Rome)**

*Line integrals of one-forms on the Sierpinski gasket*

Abstract: We give a definition of one-forms on the gasket and of their line integrals, and show that these are compatible with the notion of energy introduced by Kigami. We then introduce a suitable covering of the gasket (which is a projective limit of a sequence of natural finite coverings) and prove that n-exact forms have a primitive which lives on this covering. This is joint work with Cipriani, Guido, Sauvageot.

**Pawel Kasprzak (Copenhagen)**

*Quantum homogeneous spaces*

Abstract: The aim of this talk is to present the notion of a quantum homogeneous space. It will be shown to be an appropriate quantum counterpart of the classical notion of homogeneity. Furthermore it provides a unified framework for different classes of examples such as the quotient of a locally compact quantum group by its closed quantum subgroup due to S. Vaes, quantum homogeneous spaces of a compact quantum group studied by P. Podles and the Rieffel deformation of G-homogeneous spaces. Finally, quantum homogeneous spaces are simple objects in the category of $C^*$-algebras equipped with an action of a given quantum group.

**Yasuyuki Kawahigashi (Tokyo)**

*Superconformal field theory and operator algebras*

Abstract: We will present operator algebraic formulation of $N=1$ and $N=2$ superconformal field theories. We present basic examples, representation theory and classfication results. We will also discuss connections to noncommutative geometry, super Moonshine, and mirror symmetry of Calabi-Yau manifolds.

**Claus Köstler (Aberystwyth)**

*Symmetries and invariance principles in noncommutative probability*

Abstract: We review recent progress on de Finetti type results in noncommutative probability. Our approach shows that certain symmetries imply noncommutative conditional independence in an operator algebraic framework. Our results are applicable to subfactor theory as well as free probability. In particular they provide a new characterization of freeness with amalgamation. Moreover our de Finetti type results give a new approach to the representation theory of the infinite symmetric group. This is in parts joint work with Rolf Gohm and Roland Speicher.

**Amin Malik (Oslo)**

*On a quantization formula for a class of symplectic symmetric space*

Abstract: I will explain how one can obtain a star product on a class of local model for symplectic spaces with compatible symmetric space structure and a condition on their curvature (Ricci type), by looking for simply transitive actions and moment maps arising from groups associated to the symmetries. This is part of a joint work with M. Cahen, S. Gutt and J. Rawnsley.

**Jouko Mickelsson (Helsinki)**

*Families of Dirac operators and quantum affine algebras*

Abstract: Families of Dirac type operators constructed from the supersymmetric Wess-Zumino-Witten model are a useful tool in Fredholm operator realization of twisted K-theory classes on compact Lie groups. They transform in a covariant manner with respect to the action of a central extension of a loop group, the level of the representation giving directly the Dixmier-Douady class of the twisting gerbe. I want to describe a deformation of this system in the language of quantum affine algebras. The loop group covariance property is replaced by a "infinitesimal" Hopf algebra covariance with respect to a quantum enveloping algebra $U_q(g)$ and the Dixmier-Douady class is defined purely algebraically from the action of a central group like element in the Hopf algebra. This is an ongoing project with Antti Harju.

**Eric Morfa-Morales (ESI, Vienna)**

*Deformations of quantum field theories on spacetimes with Killing vector fields*

Abstract: I will explain how the construction and analysis of deformations of quantum field theories by warped convolutions can be extended to a class of curved spacetimes which admit two spacelike Killing fields. These spacetimes carry a family of wedge-like regions which share the essential causal properties of the Poincare transform of the Rindler wedge in Minkowski space. In the setting of deformed quantum field theories, they play the role of typical localization regions of quantum fields and observables. This is joint work with Claudio Dappiaggi (DESY Hamburg) and Gandalf Lechner (University of Vienna).

**Radu Munteanu (Bucharest)**

*Fixed point factors under product type actions on ITPFI factors*

Abstract: We consider factors arising as fixed point von Neumann algebras under product type actions of compact groups on ITPFI factors and we determine their type. In some special cases, we answer in the affirmative the following subtle question: is a fixed point factor of type $III_0$ an ITPFI factor? In order to answer this question, we identify the flow of weights of the fixed point factor with the associated flow of an ergodic hyperfinite equivalence relation and we show that it is approximately transitive, a property introduced by Connes and Woods which completely characterizes the ITPFI factors among all AFD factors. Different cases were approached by Giordano and Handelman, by using matrix-valued random walks. This is joint work with Thierry Giordano.

**Denjoe O'Connor (DIAS)**

*TBA*

Abstract:

**Eduard Ortega (Copenhagen)**

*The Cuntz semigroup and the open projection*

Abstract: We will introduce the Cuntz semigroup of a $C^*$-algebra $A$, describe its properties in certain cases and explain its usefulness for the classification program of $C^*$-algebras. We will also introduce a new picture of the Cuntz semigroup of $A$ through a certain semigroup of equivalence classes of open projections in the universal enveloping algebra of $A$. This is a joint work with Hannes Thiel and Mikael Rørdam.

**Liviu Paunescu (Rome)**

*Sofic equivalence relations and Connes' embedding problem*

Abstract: Starting from Connes' embedding problem, we shall introduce the notion of sofic equivalence relation and present known results.

**Mihai Popa (IMAR)**

*Some applications of non-commutative functions in operator-valued non-commutative probability*

Abstract: Given two vector spaces, $V$ and $W$ over the complex numbers, a non-commutative function is, briefly, a mapping from a certain class of subsets of the matrix space over $V$ to the matrix space over $W$ satisfying some compatibility conditions: it has to respect direct sums and simultaneous similarities, or equivalently, simultaneous intertwinings. Noncommutative functions admit a very nice differential calculus and they have very strong regularity properties reminiscent of the classical analytic functions, with the compatibility conditions (the respect of direct sums) playing a somewhat analogous role to the Cauchy-Riemann equations. Such objects were considered before by J. L. Taylor in his groundbreaking work on the noncommutative spectral theory, and more recently independently by D.-V. Voiculescu in free probability. The lecture will survey some applications of this theory in operator-valued non-commutative probability, such as non-commutative free Levy-Hincine formulas, Bercovici-Pata bijection, op-valued Cauchy and R-transforms, op-valued analogues of semicircle and arcsine distributions Most of the results presented are joint work with V. Vinnikov and S. Belinschi.

**Thomas Schucker (Provence)**

*Noncommutative geometry and the standard model*

Abstract: Connes' noncommutative geometry offers a beautiful way of unifying Einstein's gravity with a tiny class of Yang-Mills-Higgs models. The standard model of electro-magnetic, weak and strong forces is in this tiny class if some of its parameters meet certain constraints. The post- and predictions resulting from these constraints will be reviewed. Among these, the most striking prediction is certainly the mass of the Higgs boson at 170 +/- 10 GeV. A compilation of all theoretical predictions of the Higgs mass in the literature is also presented.

**Wojciech Szymanski (Odense)**

*Endomorphisms of Cuntz algebras*

Abstract: I will discuss recent results on endomorphisms of the Cuntz algebras which globally preserve either the core UHF subalgebra or the canonical MASA (or both). The connections with maps of the one-sided n-shift will be indicated. Time permitting, generalizations to graph algebras will be outlined.

**Hannes Thiel (Copenhagen)**

*The Chern character for low-dimensional spaces and applications*

Abstract: For every topological space there exists a ring homomorphism (called the Chern character) from K-theory to rational cohomology. This morphism becomes an isomorphism after tensoring with the rational numbers, so that K-theory and cohomology agree up to torsion. For low-dimensional spaces we can do better and can construct an isomorphism between K-theory and cohomology with integer coefficients. This has consequences for the K-theory of approximately homogeneous $C^*$-algebras (direct limits of matrices over commutative $C^*$-algebras).

**Otgonbayar Uuye (Copenhagen)**

*Homotopical algebra for C*-algebras*

Abstract: Quillen's theory of closed model categories is a powerful framework to study homotopy theory, which axiomatizes the notions of weak equivalences, fibration and cofibrations. Unfortunately, the homotopy theory of $C^*$-algebras does not fit into this framework. Fortunately, Brown demonstrated that for most purposes it is enough to have only weak equivalences and fibrations. His theory of category of fibrant objects allows one to develop homotopy theory and stable homotopy theory much the same way. In this talk, after explaining some basics about category of fibrant objects, we show that the category of $C^*$-algebras is a category of fibrant objects in a very natural way. As one consequence, we get a conceptual proof that the suspension-stable homotopy category of $C^*$-algebras is a triangulated category (which is of course well-known). Varying the notion of weak equivalences, we see that K-theory, KK-theory and E-theory also fit well in this framework.

**André Verbeure (K.U. Leuven)**

*Spontaneous Symmetry Breaking*

Abstract: Spontaneous Symmetry Breaking (SSB) has turned out to be most fruitful in explaining many modern physical phenomena. The lecture aims at presenting SSB in a mathematical setting and to prove two theorems. We explain the mathematical proof of the one-to-one relation between SSB and expectation values in two different types of physical systems. For classical spin systems the critical expectation value is the macroscopic occupation of the spin density. For interacting Bose quantum systems it is Bose-Einstein-condensation.

**Dan-Virgil Voiculescu (UCB)**

*Probability and analysis at the highest degree of noncommutativity*

Abstract: Free probability theory is a mathematical theory developed over the last 25 years which describes randomness when non-commutativity is at its highest. Parallel to a large part of classical probability the theory has models in random matrices, operator algebras and combinatorics. Related free analysis mathematical tools have begun appearing.

**Mihaly Weiner (Rome)**

*An algebraic Haag's theorem*

Abstract: Under natural conditions (such as split property and geometric modular action of wedge algebras) it is shown that the unitary equivalence class of the net of local (von Neumann) algebras in the vacuum representation associated to double cones with bases on a fixed space-like hyperplane completely determines an algebraic QFT model. More precisely, if for two models there is unitary connecting all of these algebras, then - without assuming that this unitary also connects their respective vacuum states or spacetime symmetry representations - it follows that the two models are equivalent. This new result might be regarded as an algebraic version of the celebrated theorem of Rudolf Haag which showed that there are some problems with the so-called "interaction-picture". Original motivation of the speaker for finding such an algebraic version came from conformal chiral QFT. Both the chiral case as well as a related conjecture about standard half-sided modular inclusions will be also discussed.

**Bora Yalkinoglu (Paris VI)**

*On Bost-Connes type systems and Complex Multiplication*

Abstract:

**Jakob Yngvason (ESI, Vienna)**

*Quantum fields localized in cones and wedges*

Abstract: In the setting of algebraic quantum field theory the basic mathematical objects are nets of operator algebras indexed by the regions of space-time where the corresponding operators are assumed to act. Usually there is a nontrivial algebra associated with any open region of space-time but there are situations where weaker localization properties arise naturally. In particular, the concept of modular localization as well as that of gauge charges leads to localization in space-like cones, and recent ideas about deformations of nets of operator algebras have produced new models where the natural localization regions are space-like wedges. In the lecture a survey of ideas and results about such nets with weakened localization properties will be presented.

**Laszlo Zsido (Rome)**

*Relative weakly mixing for W*-dynamical systems*

Abstract: Weakly mixing and almost periodicity are opposite recurrence properties of automorphisms (or more generally, automorphism groups) of von Neumann algebras, leaving invariant a faithful normal state. Weakly mixing can also be defined relative to an invariant von Neuman subalgebra and we intend to discuss thoroughly relative weakly mixing. Given an arbitrary automorphism of a von Neumann algebra $M$, which preserves some faithful normal state of $M$, we shall also discuss the existence of an invariant von Neuman subalgebra $N$ of $M$, relative to which the given automorphism is weakly mixing and such that the restriction of the automorphism to $N$ is almost periodic.