

# GENERALIZED RENEWAL PROCESSES AND PSEUDO-REGULARLY VARYING FUNCTIONS

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Let  $\{X_k\}$  be a sequence of independent identically distributed random variables and  $\{S_n\}$  and  $\{N_t\}$  be the sequence of their cumulative sums and renewal process constructed from  $\{S_n\}$ . It is a point of view in [1] that  $\{N_t\}$  and  $\{S_n\}$  are “inverses” each to other. Another idea presented in [1] is that properties of  $\{N_t\}$  “follow” from the corresponding properties of  $\{S_n\}$  and *vice versa*.

Generally, we call two objects *dual* if they are “inverse” to each other in some sense and their asymptotic properties are related to each other, that is, a limit result for the first object implies a corresponding one for the second object, and *vice versa*. We provide several examples of dual objects in mathematics in the talk.

The process  $N_t$  is the *first exit time* when  $\{S_n\}$  exists from  $(0, t)$ . If random variables  $\{X_k\}$  are nonnegative, then it is the *last exit time* as well as the *sojourn time*. But if values of the underlying random variables  $\{X_k\}$  are of both signs, then the above three functionals are different. According to [1], they nevertheless are *generalized renewal processes*.

We discuss the question on conditions being sufficient for all three functionals to be asymptotically equivalent and thus to be dual objects to  $\{S_n\}$ . The conditions are expressed in terms of *pseudo-regularly* functions. This notion is related to Karamata’s regular variation.

The case of dependent or non-identically distributed  $\{X_k\}$  can also be treated.

## REFERENCES

- [1] V. V. Buldygin, K.-H. Indlekofer, O. I. Klesov, and J. G. Steinebach *Pseudo-Regularly Varying Functions and Generalized Renewal Processes*, TBiMC, Kiev, 2012, 442 pp. (in Ukrainian)

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