GENERALIZED RENEWAL PROCESSES AND PSEUDO-REGULARLY VARYING FUNCTIONS

O. I. KLESOV

Let \( \{X_k\} \) be a sequence of independent identically distributed random variables and \( \{S_n\} \) and \( \{N_t\} \) be the sequence of their cumulative sums and renewal process constructed from \( \{S_n\} \). It is a point of view in [1] that \( \{N_t\} \) and \( \{S_n\} \) are “inverses” each to other. Another idea presented in [1] is that properties of \( \{N_t\} \) “follow” from the corresponding properties of \( \{S_n\} \) and \textit{vice versa}.

Generally, we call two objects \textit{dual} if they are “inverse” to each other in some sense and their asymptotic properties are related to each other, that is, a limit result for the first object implies a corresponding one for the second object, and \textit{vice versa}. We provide several examples of dual objects in mathematics in the talk.

The process \( N_t \) is the \textit{first exit time} when \( \{S_n\} \) exists from \((0,t)\). If random variables \( \{X_k\} \) are nonnegative, then it is the \textit{last exit time} as well as the \textit{sojourn time}. But if values of the underlying random variables \( \{X_k\} \) are of both signs, then the above three functionals are different. According to [1], they nevertheless are \textit{generalized renewal processes}.

We discuss the question on conditions being sufficient for all three functionals to be asymptotically equivalent and thus to be dual objects to \( \{S_n\} \). The conditions are expressed in terms of \textit{pseudo-regularly} functions. This notion is related to Karamata’s regular variation.

The case of dependent or non-identically distributed \( \{X_k\} \) can also be treated.

References


Department of Mathematical Analysis and Probability Theory, National Technical University of Ukraine “Kyiv Polytechnic Institute”, 37 Peremogy Avenue, 03056 Kyiv, Ukraine

\textit{E-mail address}: klesov@matan.kpi.ua